CALCULUS III NAME _____

EXAM I Rec. Instr. _____

FALL 1997 Rec. Time _____

TO RECEIVE CREDIT YOU MUST SHOW YOUR WORK.

(25) 1. An object is moving in 3-space according to the parametric equations $x = \cos t$, $y = \sin t$ and $z = 3\sin t$ where t is the time in seconds. Find, as functions of t,

a) position vector $\vec{r} =$

b) velocity vector $\vec{v} =$

c) acceleration vector $\vec{a} =$

d) speed $\dfrac{ds}{dt} =$

e) tangential component of acceleration $a_T =$

f) curvature $\kappa =$

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(15) 2. A particle is moving in the plane. Its acceleration vector as a function of time is $\vec{a} = (\sin t)\vec{i} - 3\vec{j}$. Suppose that at $t = 0$ its velocity vector is $\vec{v}(0) = \vec{j}$ and its position vector is $\vec{r}(0) = \vec{i} - \vec{j}$. Find the velocity vector and the position vector as functions of t. Then write the parametric equations for the motion.

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(15) 3. A particle is moving in 3-space. Suppose you know that at $t = 2$ seconds the position vector is $\vec{r}(2) = 2\vec{i} + \vec{j} - \vec{k}$, the velocity vector is $\vec{v}(2) = \vec{i} + 2\vec{j} + 2\vec{k}$ and the acceleration vector is $\vec{a}(2) = 3\vec{i} - 4\vec{j}$. Using this information only, answer the following questions. Do not attempt to find $\vec{r}$, $\vec{v}$ and $\vec{a}$ as functions of t. Assume distance is measured in feet.

At $t = 2$ seconds

a) at which point is the particle located?

b) what is the speed?

c) what is the unit tangent vector $\vec{T}$?

d) what is the value of a_T?

e) what is the value of a_N?

f) is it speeding up or slowing down? Why?

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(15) 4. A particle is moving in the plane along the curve $y = \frac{5}{2} - \frac{1}{2}x^2$. It is moving from left to right at a constant speed of 3 ft/sec.

a) Find a_T and a_N at the point $\left(x, \frac{5}{2} - \frac{1}{2}x^2\right)$.

b) Find the velocity vector and acceleration vector when the particle is at the point $(1, 2)$.

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(15) 5. Given the 3-dimensional vectors

$$\begin{aligned} \vec{a} &= 3\vec{i} - \vec{j} + 3\vec{k} \\ \vec{b} &= \vec{i} + \vec{j} - 2\vec{k} \\ \vec{c} &= 4\vec{i} + \vec{j} - 2\vec{k} \end{aligned}$$

Find

a) the cosine of the angle between $\vec{a}$ and $\vec{b}$

b) $\vec{a} \times \vec{b}$

c) the area of the parallelogram determined by $\vec{a}$ and $\vec{b}$

d) the volume of the parallelepiped determined by $\vec{a}$, $\vec{b}$ and $\vec{c}$.

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(15) 6. Given the points $P(1,-1,4)$, $Q(3,1,5)$ and $R(1,2,3)$

Find

a) a unit vector in the direction of $\overrightarrow{PQ}$

b) parametric equations for the line through P and R

c) the equation of the plane containing the points P, Q and R

d) the area of the triangle with vertices P, Q and R