CALCULUS III NAME _____

EXAM I Rec. Instr. _____

SPRING 2000 Rec. Time _____

TO RECEIVE CREDIT YOU MUST SHOW YOUR WORK.

(20) 1. Given vectors

$$\vec{a} = 2\vec{i} + \vec{j} - \vec{k}$$
$$\vec{b} = \vec{i} + \vec{j} + 5\vec{k}$$
$$\vec{c} = \vec{i} + 2\vec{j} + \vec{k}$$

find

a) the angle between $\vec{a}$ and $\vec{b}$.

b) the area of the parallelogram determined by $\vec{a}$ and $\vec{b}$ when they are placed with the same initial point.

c) the volume of the parallelepiped determined by $\vec{a}$, $\vec{b}$ and $\vec{c}$ when they are placed with the same initial point.

(15) 2.a) Find the equation of the plane which contains the point $P_o(1, 3, -2)$ and is perpendicular to the line

$$x = 2 - 3t, \quad y = 1 + t, \quad z = 4 + 4t$$

b) Find the equation of the plane which passes through the points $A(1, 0, 1)$, $B(2, 3, 1)$, $C(1, 2, 2)$.

(15) 3. An object is moving in 3-space according to the parametric equations $x = 1 + t$, $y = 2t$, $z = \sin t$. Find, as functions of the time t,

a) position vector $\vec{r} =$

b) velocity vector $\vec{v} =$

c) acceleration vector $\vec{a} =$

d) speed $\dfrac{ds}{dt} =$

e) tangential component of acceleration $a_T =$

f) the curvature $\kappa =$

g) the normal component of acceleration $a_N =$

(15) 4. An object is moving in 3-space in such a way that its acceleration vector as a function of the time t is $\vec{a} = (\sin t)\vec{k}$. Suppose you know that at time $t = 0$, the velocity vector is $\vec{v}(0) = \vec{i} + \vec{j}$ and the position vector is $\vec{r}(0) = \vec{j}$. Find the velocity vector and the position vector as functions of the time t and then give the parametric equations of the motion.

(15) 5. An object is moving in the xy-plane along the curve $y = \frac{5}{2} - \frac{1}{2}x^2$ in the direction of increasing x. It is moving at a constant speed of 5 ft/sec.

a) Find a_T and a_N at the general point $\left(x, \frac{5}{2} - \frac{1}{2}x^2\right)$.

b) Find the velocity vector and the acceleration vector when the object is at the point $(1, 2)$.

(20) 6. Let $f(x, y) = xy^2 + x^3y + \dfrac{5y^3}{2y}$.

a) Calculate all 1st and 2nd order partial derivatives of $f(x, y)$.

b) Find all points on the surface $z = f(x, y)$ at which the tangent plane is horizontal.