CALCULUS III NAME _____

EXAM Rec. Instr. _____

SPRING 1999 Rec. Time _____

TO RECEIVE CREDIT YOU MUST SHOW YOUR WORK.

(15) 1. An object is moving in 3-space in such a way that its acceleration vector as a function of time is $\vec{a} = (\cos t)\vec{i} - (\sin t)\vec{k}$. Suppose you know that at time $t = 0$, the velocity vector is $\vec{v}(0) = \vec{i} + \vec{j} + \vec{k}$ and the position vector is $\vec{r}(0) = \vec{j}$. Find the velocity vector and the position vector as functions of t, and then give the parametric equations of the motion.

NAME _____

Rec. Instr. _____

(15) 2. A particle is moving in the plane along the curve $y = (x+1)^2$ in the direction of increasing x. It is moving at a constant speed of 4 ft/sec.

a) Find a_T and a_N when the particle is at $(x, (x+1)^2)$.

b) Find the velocity vector and the acceleration vector when the particle is at $(0, 1)$.

NAME _____

Rec. Instr. _____

(15) 3. The force field $\vec{F} = -y\vec{i} + 2x\vec{j}$ acts on an object as it moves in the xy-plane. Calculate the work done by $\vec{F}$ as the object moves from $(0,0)$ to $(2,7)$ along the following path:

a) First, along $y = x^2$ from $(0,0)$ to $(2,4)$.

b) Second, along the vertical line segment from $(2,4)$ to $(2,7)$.

NAME _____

Rec. Instr. _____

(15) 4. Let $f(x, y) = x^3 + xy^2 - xy$.

a) If you are moving along a straight line throught the point $(x, y) = (1, 2)$ toward the point $(x, y) = (4, 1)$ what is the instantaneous rate of change of $f(x, y)$ per unit distance that you will observe at $(1, 2)$?

b) If you are at $(x, y) = (1, 2)$, in which direction should you go if you want $f(x, y)$ to increase most rapidly?

NAME _____

Rec. Instr. _____

(20) 5. Find the equation of the tangent plane to each of the following surfaces at the specified point

a) to the graph of $f(x, y) = xe^{-xy}$ at $(1, -2, e^2)$.

b) to $xy^2 + yz^2 + zx^2 = 5$ at $(1, 2, -1)$

c) to the parametrized surface $x = s^2 + t$, $y = t^2 + s$, $z = s^3$ at the point obtained when $s = 2$ and $t = 1$.

NAME _____

Rec. Instr. _____

(20) 6. Find and classify the critical points of $f(x, y) = y^2 - x^3y + 3xy$.

NAME _____

Rec. Instr. _____

(15) 7. Use the method of Lagrange multipliers to find the largest value and the smallest value of $f(x, y) = x^2 + y^2$ on the curve $x^4 + y^4 = 16$.

NAME _____

Rec. Instr. _____

(15) 8. Find the volume of the 3-D region enclosed by the surfaces $y = x^2$, $y = 4$, $z = 5 + x$, $z = 2x$.

(10) 9. A mass distribution occupies the region which is above $z = x^2 + y^2$ and under $z = 4$. If the mass density function is $\delta(x, y, z) = z^2$ units of mass/unit volume, calculate the total mass.

NAME _____

Rec. Instr. _____

(10) 10. Show that the force field

$$\vec{F} = \left(2xy + \frac{2x}{(x^2+y^2)}\right)\vec{i} + \left(x^2 + \frac{2y}{(x^2+y^2)} + 1\right)\vec{j}$$

is conservative in the region $(x, y) \neq (0, 0)$ by finding a potential function for it. Now use this potential function to calculate the work done by $\vec{F}$ as it acts on an object which moves from $(2, 1)$ to $(3, 4)$ along any path which does not pass through $(0, 0)$.

NAME _____

Rec. Instr. _____

(10) 11. Use Green's theorem to evaluate the line integral $\int_C (y^2 + 12xy)\,dx + (6x^2 + y^2)\,dy$ where C is the triangle with vertices $(0,0), (1,0), (0,1)$ directed counterclockwise.

(10) 12. Evaluate the surface integral $\int\int_S y\,dS$ where S is the parametrized surface $x = \frac{1}{2}t^2\,,\ y = 2t\,,\ z = s\,,\ 0 \le s \le 2\,,\ 0 \le t \le 1\,.$

NAME _____

Rec. Instr. _____

(15) 13. Verify the divergence theorem (page 940) for the special case where the vector field is $\vec{F} = z^3\vec{k}$ and the surface S is the sphere $x^2 + y^2 + z^2 = 1$ by direct calculation of both the surface integral and the triple integral.

Hint: To do the surface integral parametrize the sphere using spherical coordinates with $\rho = 1$. Then $dS = \sin\phi \; d\phi \; d\theta$.

NAME _____

Rec. Instr. _____

(15) 14. Let $\vec{F}$ be the vector field $\vec{F} = 2yz\,\vec{i} + xy\,\vec{k}$.

a) $curl\vec{F} = \nabla \times \vec{F} =$

b) Let S be the surface $z = 4 - x^2 - y^2$, $z \geq 0$. For this surface and the given vector field, verify Stokes' theorem (page 947) by directly calculating the value of both the surface integral and the line integral.