CALCULUS III NAME _____

EXAM Rec. Instr. _____

SPRING 1998 Rec. Time _____

TO RECEIVE CREDIT YOU MUST SHOW YOUR WORK.

(20) 1. An object is moving in 3-space in such a way that its acceleration vector as a function of time is $\vec{a}(t) = (-\sin t)\vec{k}$. At time $t = 0$ its velocity vector is $\vec{v}(0) = \vec{i} + \vec{j}$ and its position vector is $\vec{r}(0) = \vec{i}$.

a) Find the velocity vector and the position vector as functions of the time t.

b) Write the parametric equations for the motion.

c) Find a_N, the normal component of acceleration as a function of time.

NAME _____

Rec. Instr. _____

(15) 2. An object is moving around the ellipse $x^2 + 144y^2 = 144$ in the plane. It is moving at a constant speed of 3 ft/sec.

a) Specify which of the following statements is (are) true and explain why.

i) At each instant the acceleration vector is parallel to the velocity vector.

ii) The acceleration vector is zero.

iii) At each instant the acceleration vector is perpendicular to the velocity vector.

b) Find the points on the ellipse at which a_N takes on its largest value and the points at which it takes on its smallest value.

c) Determine a_T as a function of time.

NAME _____

Rec. Instr. _____

(10) 3. Show that the force field $\vec{F} = \dfrac{x}{x^2+y^2}\vec{i} + \dfrac{y}{x^2+y^2}\vec{j}$ is conservative in the region of the plane for which $(x,y) \neq (0,0)$ by finding a potential function for it. Then use this potential function to calculate the work done by $\vec{F}$ as it acts on an object which moves from the point (4,3) to the point (-2,1) along a path which does not pass through the origin.

(15) 4. The tornado force field $\vec{F} = \dfrac{-y}{(x^2+y^2)^2}\vec{i} + \dfrac{x}{(x^2+y^2)^2}\vec{j}$ acts on an object as it moves in the plane. Calculate the work done by $\vec{F}$ as the object moves along the upper half of the circle $x^2+y^2=a^2$ from $(a,0)$ to $(-a,0)$. What happens as you take a to be smaller and smaller?

NAME _____

Rec. Instr. _____

(10) 5. Use Green's theorem to evaluate the line integral $\int_C (x^2 + y^2)dx + (4xy - y^2)dy$ where C is the closed curve made up of the sides of the triangle with vertices $(0,0)$, $(1,0)$ and $(1,1)$ directed counterclockwise.

(15) 6. A parametric surface is given by $x = s\cos t$, $y = s\sin t$, $z = t$. Calculate the area of that part of the surface given by $0 \le s \le 1$, $0 \le t \le 2\pi$.

NAME _____

Rec. Instr. _____

(15) 7. A mass distribution occupies the region in 3-space which is above the surface $z = \sqrt{x^2 + y^2}$ and under the plane $z = 4$. The mass density function is $\delta(x, y, z) = z$ units of mass/unit volume. Use a triple integral in spherical coordinates to calculate the total mass.

NAME _____

Rec. Instr. _____

(15) 8. Calculate the volume of the 3-D region which is bounded by the surfaces $z = 4 + x^2$, $x^2 + y^2 = 4$ and $z = 0$. Use polar coordinates to evaluate the double integral.

NAME _____

Rec. Instr. _____

(20) 9. Let $f(x,y) = (y^2-1)\cos x + \frac{1}{3}y^3$. Find and classify those critical points of f which are in the region of the xy-plane for which $-\frac{\pi}{2} < x < \frac{\pi}{2}$.

NAME _____

Rec. Instr. _____

(20) 10. Let $f(x, y) = x\sqrt{x^2 + y^2}$.

a) Find the gradient vector field of f.

b) Find the instantaneous rate of change of f at $(3, 4)$ which you would observe if you moved along a straight line through $(3, 4)$ towards $(4, 1)$.

c) Find the direction in which you should proceed from $(3, 4)$ if you wish to obtain the most rapid increase possible for f.

NAME _____

Rec. Instr. _____

(25) 11. Find equations for the following planes.

a) Plane tangent to $z^3 + xz^2 + x^3y + y^3 = 12$ at $(1, 2, 1)$.

b) Plane through the points $(1, 0, 2)$, $(2, 1, 3)$ and $(3, 1, 2)$.

c) Plane tangent to the graph of $f(x, y) = xe^{xy}$ at $(1, 1, e)$.

NAME _____

Rec. Instr. _____

(20) 12. Given the four points in 3-space $P_1(2,0,1)$, $P_2(3,1,1)$, $P_3(3,4,3)$, $P_4(4,1,2)$

a) Find the angle between $\overrightarrow{P_1P_2}$ and $\overrightarrow{P_1P_3}$.

b) Find the area of the triangle with vertices P_1, P_2 and P_3.

c) Find the volume of the parallelepiped determined by $\overrightarrow{P_1P_2}$, $\overrightarrow{P_1P_3}$ and $\overrightarrow{P_1P_4}$