NAME _____

CALCULUS III EXAM Rec. Instr. _____

SPRING 2003 Rec. Time _____

TO RECEIVE CREDIT YOU MUST SHOW YOUR WORK.

(20) 1) The points $P(1,0,2)$, $Q(2,1,2)$ and $R(0,-2,4)$ are vertices of a triangle in 3-space.

a) Find the angle, in degrees, at the vertex P.

b) Find the area of the triangle.

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(20) 2. Find the equation for each of the following planes.

a) The plane containing the points P, Q and R of problem 1.

b) The tangent plane to the surface $x^2 + z^4 = 5y^2 + 5z^2$ at the point $(3, 1, 2)$.

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(20) 3) Let $f(x, y, z) = xz^2 + yx^2 + zy^2$.

a) Find the directional derivative of $f(x, y, z)$ at $(x, y, z) = (1, 2, -1)$ in the direction of the vector $\vec{a} = \vec{i} + 5\,\vec{j} + 3\,\vec{k}$.

b) Find the value of the largest directional derivative of $f(x, y, z)$ at $(1, 2, -1)$. In which direction does it occur?

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(20) 4) An object is moving in 3-space in such a way that its acceleration vector as a function of the time t is

$$\vec{a} = (\cos t + \sin t)\,\vec{i} + (\cos t - \sin t)\,\vec{j}.$$

Suppose that at time $t = 0$ its velocity vector is $\vec{v}(0) = -\vec{i} + \vec{j} + 2k$. Find, as functions of t, a_T, a_N, curvature, $\vec{T}$ and $\vec{N}$.

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(20) 5) Let $f(x, y) = -x + xy - (e^y)(x^{-1})$. Find and classify the critical points of $f(x, y)$ in the region $x \neq 0$.

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(15) 6) Use the method of LAGRANGE MULTIPLIERS to find the smallest value of $F(x, y, z) = x^2 + y^2 + z^4$ on the surface $xyz = 64$ for $x > 0$, $y > 0$ and $z > 0$.

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(10) 7) Calculate the volume of the 3D-region in the first octant which is enclosed by the surfaces $y = x$, $y = 2$, $x = 0$, $z = 2y$ and $z = 6$.

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(15) 8) The force field $\vec{F} = xy\,\vec{i} - x\,\vec{j}$ acts on an object as it moves in the plane. Find the work done by $\vec{F}$ as the object moves from $(2, 0)$, to $(-2, 0)$ along the upper half of the circle $x^2 + y^2 = 4$.

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(10) 9) Show that the force field $\vec{F} = (2x + y^{-1})\,\vec{i} + (2y - xy^{-1})\,\vec{j}$ is conservative in the region $y > 0$ by finding a potential function for it. Now use this potential function to calculate the work done by $\vec{F}$ as it acts on an object which moves from $(-1, 1)$ to $(4, 2)$ along any curve in the upper half plane.

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(15) 10) Use Green's Theorem to evaluate the line integral

$$\int_C (3y^2 + 3x^2 - y^3)dx + (6xy + x^3)dy$$

where C is the curve consisting of the x-axis from $(-2, 0)$ to $(2, 0)$ followed by the upper half of the circle $x^2 + y^2 = 4$ from $(2, 0)$ to $(-2, 0)$.

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(15) 11) Let T be the 3D-region which is inside the sphere $x^2+y^2+z^2=4$ and above the xy-plane, and S be its boundary. If $\vec{F} = xz\,\vec{i} + yz\,\vec{j} + z\,\vec{k}$, use the divergence theorem to find the value of the outward flux integral of $\vec{F}$ across S.

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(20) 12) Let S be that part of $z = x^2 + y^2$ which is in the first octant and under the plane $z = 4$. Suppose that $\vec{F} = y\,\vec{i} - x\,\vec{j} + y^2\,\vec{k}$.

a) Calculate curl $\vec{F}$.

b) Let C be the positively oriented boundary of S where S is given the upward pointing normal. Use Stokes' Theorem to find the value of the line integral $\displaystyle\int_C \vec{F} \times \vec{T} ds$.