CALCULUS III NAME _____

FINAL EXAM Rec. Instr. _____

SPRING 2002 Rec. Time _____

TO RECEIVE CREDIT YOU MUST SHOW YOUR WORK.

(25) 1) An object is moving in 3-space according to the parametric equations $x = t$, $y = t^2$, $z = 2t$. Find as functions of the time t,

a) the position vector $\vec{r} =$

b) the velocity vector $\vec{v} =$

c) the acceleration vector $\vec{a} =$

d) the speed $\dfrac{ds}{dt} =$

e) the tangential component of acceleraton $a_T =$

f) the curvature $\kappa =$

g) the normal component of acceleration $a_N =$

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(15) 2) An object is moving in 3-space in such a way that its acceleration vector as a function of the time is $\vec{a} = e^{-t}\,\vec{i} + \vec{k}$. Suppose that at time $t = 0$ you know that the velocity vector is $\vec{v}(0) = \vec{j}$ and the position vector $\vec{r}(0) = \vec{i} + \vec{k}$. Find the velocity vector and the position vector as functions of t and then give the parametric equations of the motion.

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(15) 3) A quantity Q depends upon x and y according to $Q = x\sqrt{y^2 + 4x}$. Suppose x and y are changing with time t and at a certain instant you know that $x = 2$, $y = 1$, $\frac{dx}{dt} = -1$ and $\frac{dy}{dt} = 2$. Use the chain rule to find $\frac{dQ}{dt}$ at this instant.

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(10) 4) Find the equation of the tangent plane to the surface $(x+z)e^{yz} = 6e^{-2}$ at the point $(x, y, z) = (4, -1, 2)$.

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(25) 5. Find and classify the crititcal points of $f(x, y) = y^3 - x^3y + 12xy$.

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(15) 6. Find the volume of the 3d-region enclosed by the surfaces $y = x^2$, $y = 4$, $z = y$ and $z = 12$.

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(20) 7. The force field $\vec{F} = y\vec{i} + (xy+1)\vec{j}$ acts on an object as it moves in the plane. Calculate the work done by $\vec{F}$ for each of the following motions.

a) The object moves from $(0,0)$ to $(4,2)$ along the curve $x = y^2$.

b) The object moves from $(0,0)$ to $(4,2)$ by going first along the y-axis from $(0,0)$ to $(0,2)$ and then along the line segment from $(0,2)$ to $(4,2)$.

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(10) 8. Show that the force field

$$\vec{F} = (2x + \ln y)\vec{i} + \left(1 + \frac{x}{y}\right)\vec{j}$$

is conservative in the region $y > 0$ by finding a potential function for it. Now use this potential function to calculate the work done by $\vec{F}$ as it acts on an object which moves in the upper half plane from $(-1, 1)$ to $(3, 2)$.

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(15) 9. Use Green's theorem to evaluate

$$\int_C (6y + 2x^3y^2)\,dx + (2x + x^4y)\,dy$$

where C is the circle $(x-1)^2+(y-5)^2 = 9$ directed counterclockwise.

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(15) 10. Let S be the parametrized surface $x = s + t$, $y = s - t$, $z = s + \frac{2}{3}t^{\frac{3}{2}}$, $0 \leq s \leq 2$, $0 \leq t \leq 1$. Evaluate the surface integral $\int\int_S (x + y)\, dS$.

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(15) 11. <u>Use the divergence theorem</u> to evaluate $\int\int_S \vec{F} \cdot \vec{n}\, dS$ where $\vec{F} = xy^2\,\vec{i} + yz^2\,\vec{j} + zx^2\,\vec{k}$ and S is the sphere $x^2 + y^2 + z^2 = 4$ with $\vec{n}$ the outward pointing unit normal.

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(20) 12. Let S be that part of the paraboloid $z = 9 - x^2 - y^2$ which is in the first octant oriented with the upward pointing normal and let C be the positively oriented boundary of S. If $\vec{F} = (-yz)\,\vec{i} + (xz + x)\,\vec{j} + \vec{k}$ <u>use Stokes' theorem</u> to evaluate

$$\int_C \vec{F} \cdot \vec{T}\, ds = \int_C -yz\, dx + (xz + x)\, dy + dz.$$