CALCULUS III NAME ____________________

EXAM Rec. Instr. ____________________

SPRING 2001 Rec. Time ____________________

TO RECEIVE CREDIT YOU MUST SHOW YOUR WORK.

(20) 1. An object is moving in 3-space and its acceleration vector as a function of time is $\vec{a} = (\cos t)\,\vec{j} - (\sin t)\,\vec{k}$. Suppose that at time $t = 0$ the velocity vector is $\vec{v}(0) = \vec{i} + \vec{k}$ and the position vector $\vec{r}(0) = \vec{i}$.

a) Find the velocity vector as a function of time.

b) Find the position vector as a function of time.

c) Find the curvature as a function of time.

d) Give the parametric equations of the motion.

NAME ____________________

Rec. Instr. ____________________

(10) 2. An object is moving around the circle $x^2 + y^2 = 4$. It starts at time $t = 0$ and its speed as a function of time t is $\frac{ds}{dt} = t^2$.

a) Find the tangential component of acceleration as a function of time.

b) Find the normal component of acceleration as a function of time.

NAME ______________________

Rec. Instr. ______________________

(15) 3. The force field $\vec{F} = -2y\,\vec{i} + xy\,\vec{j}$ acts on an object as it moves in the plane. Calculate the work done by $\vec{F}$ as the object moves from $(0,0)$ to $(4,-3)$ by going first along $x = y^2$ from $(0,0), (4,2)$ and then along the vertical line segment from $(4,2)$ to $(4,-3)$.

NAME ____________________

Rec. Instr. ____________________

(15) 4. Let $f(x, y) = y^3 + x^2y + xy$.

a) If you are moving along a straight line through the point $(x, y) = (1, 2)$ toward the point $(x, y) = (3, 1)$ what is the instantaneous rate of change of $f(x, y)$ per unit distance that you will observe at $(x, y) = (1, 2)$?

b) If you are at $(x, y) = (1, 2)$, in which direction should you go if you want $f(x, y)$ to increase most rapidly?

NAME ______________________________

Rec. Instr. ______________________________

(20) 5. Find the equation of the tangent plane to each of the following surfaces at the specified point

a) to the graph of $f(x,y) = ye^{-xy}$ at $(1,-1,-e)$

b) to $zy^2 + yx^2 + xz^2 = 5$ at $(-1,2,1)$

NAME ______________________________

Rec. Instr. ______________________________

(15) 6. Suppose $z = f(x, y)$ and we know that at $(x, y) = (2, 1)$ we have $\dfrac{\partial f}{\partial x}(2, 1) = -3$ and $\dfrac{\partial f}{\partial y}(2, 1) = 2$. If (r, θ) denote the usual polar coordinates in the xy-plane use the chain rule to calculate $\dfrac{\partial z}{\partial r}$ and $\dfrac{\partial z}{\partial \theta}$ at the point $(x, y) = (2, 1)$.

NAME ______________________________

Rec. Instr. ______________________________

(15) 7. Use the method of Lagrange multipliers to find the maximum value and the minimum value of $f(x, y) = z^2 + xy$ on the sphere $x^2 + y^2 + z^2 = 8$.

NAME ______________________________

Rec. Instr. ______________________________

(15) 8. A mass distribution occupies the 3D-region enclosed by the cone $z = \sqrt{x^2 + y^2}$ and the plane $z = 4$. The mass density function is $\delta(x, y, z) = x^2 + y^2 + z^2$ units of mass/unit volume. Use a triple integral in spherical coordinates to calculate the total mass.

NAME ______________________

Rec. Instr. ______________________

(15) 9. Calculate the volume of the 3D-region enclosed by the surfaces $y = x^2$, $y = 4$, $z = 2$ and $z = 4 + y$.

(10) 10. Evaluate the double integral $\int\int_R (x + y)dA$ where R is the region in the 1st quadrant of the xy-plane enclosed by $y = x$, $x^2 + y^2 = 4$ and $y = 0$.

NAME ______________________

Rec. Instr. ______________________

(10) 11. Evaluate the surface integral $\int\int_S xz\,dS$ where S is the parametrized surface $x = t$, $y = \frac{1}{2}t^2$, $z = s^2$, $1 \leq s \leq 2$, $0 \leq t \leq 2$.

NAME ____________________

Rec. Instr. ____________________

(10) 12. Show that the force field

$$\vec{F} = \left(2x + \frac{1}{y}\right)\vec{i} + \left(3y^2 - \frac{x}{y^2}\right)\vec{j}$$

is conservative in the region $y > 0$ by finding a potential function for it. Now calculate the work done by $\vec{F}$ as it acts on an object which moves in the upper half plane from $(0, 3)$ to $(6, 2)$.

(10) 13. Use Green's theorem to evaluate the line integral

$$\int_C (\sin x + xy^2 + 4y)dx + (x^2y + e^y)dy$$

where C is the circle $(x-1)^2 + y^2 = 16$ directed counter clockwise.

NAME ______________________

Rec. Instr. ______________________

(20) 14. Verify the divergence theorem (page 1004) where S is the closed surface consisting of two pieces,

a) the part of the paraboloid $z = x^2 + y^2$ for $0 \leq z \leq 4$

b) the disk, $x^2 + y^2 \leq 4$, $z = 4$

and $\vec{F} = x\,\vec{i} + y\,\vec{j} + z\vec{k}$.