CALCULUS III NAME _____

FINAL EXAM Rec. Instr. _____

SPRING 2000 Rec. Time _____

TO RECEIVE CREDIT YOU MUST SHOW YOUR WORK.

(20) 1. An object is moving in 3-space according to the parametric equations $x = \cos t$, $y = t^2$, $z = \sin t$. Find, as functions of t,

a) position vector $\vec{r} =$

b) velocity vector $\vec{v} =$

c) acceleration vector $\vec{a} =$

d) speed $\dfrac{ds}{dt} =$

e) tangential component of acceleration $a_T =$

f) normal component of acceleration $a_N =$

g) curvature $k =$

h) the cosine of the angle between $\vec{v}$ and $\vec{a}$.

NAME _____

Rec. Instr. _____

(15) 2. An object is moving in 3-space in such a way that its acceleration vector as a function of time t is $\vec{a} = \vec{j} + (\cos t)\vec{k}$. Suppose that at time $t = 0$ you know that its velocity vector is $\vec{v}(0) = \vec{i} + \vec{j}$ and its position vector is $\vec{r}(0) = \vec{i} + \vec{k}$. Find the velocity vector and the position vector as functions of the time t. Now give the parametric equations for its motion.

NAME _____

Rec. Instr. _____

(10) 3. Find the directional derivative of $f(x, y) = xe^{xy}$ at the point $(x, y) = (2, 3)$ in the direction of the vector $\vec{a} = -2\vec{i} + \vec{j}$.

NAME _____

Rec. Instr. _____

(15) 4. Find the equation for each of the following planes.

a) Tangent plane to the surface $x^2z^3 + xyz + y^3 = 23$ at the point $(x, y, z) = (3, 2, 1)$.

b) Plane through the points $(1, 1, 1)$, $(2, 1, 0)$ and $(1, 0, 3)$.

NAME _____

Rec. Instr. _____

(15) 5. A quantity Q depends upon x and y according to $Q = \dfrac{x}{x^2+y^2}$. Suppose that x and y are changing with time and at a certain instant you know that $x = 1$, $y = 2$, $\dfrac{dx}{dt} = 2$ and $\dfrac{dy}{dt} = 4$. Use the Chain Rule to find $\dfrac{dQ}{dt}$ at this instant.

NAME _____

Rec. Instr. _____

(20) 6. Use the method of Lagrange multipliers to find the largest value and the smallest value of $f(x, y, z) = x + yz$ on the ellipsoid $4x^2 + 4y^2 + z^2 = 9$.

NAME _____

Rec. Instr. _____

(25) 7. Find and classify the critical points of $f(x, y) = xy^2 + x^3 - 2x^2 - y^2$.

NAME _____

Rec. Instr. _____

(10) 8. Use Green's theorem to evaluate the line integral

$$\int_C (2xy + ye^x + 3y)dx + (x^2 + e^x)dy$$

where C is the circle $(x-1)^2 + (y-3)^2 = 9$ directed counterclockwise.

(15) 9. The force field $\vec{F} = (x+y)\vec{i} + 2x\vec{j}$ acts on an object as it moves in the plane. Calculate the work done by $\vec{F}$ as the object moves from $(0,0)$ to $(6,2)$ by going first along the curve $x = y^2$ from $(0,0)$ to $(4,2)$ and then along the line segment from $(4,2)$ to $(6,2)$.

NAME _____

Rec. Instr. _____

(15) 10. Evaluate the surface integral $\int\int_S (y+z)dS$ where S is the parametrized surface $x = u^2$, $y = u - v$, $z = u + v$, $0 \le u \le 1$, $0 \le v \le 2$.

NAME _____

Rec. Instr. _____

(20) 11. Let S be the closed surface, consisting of two pieces, which bounds the 3-dimensional region which is under $z = 4 - x^2 - y^2$ and above the xy-plane. Suppose $\vec{F} = x\vec{i} + y\vec{j} + (z+1)\vec{k}$. Verify the Divergence Theorem in this case by calculating both sides of the equation and seeing that they are equal.

NAME _____

Rec. Instr. _____

(20) 12. Verify Stokes' Theorem by directly calculating both sides of the equation in the special case where the vector field is $\vec{F} = zy\vec{i} + z\vec{k}$ and the surface is $z = x^2 + y^2\,,\ z \leq 4\,.$