CALCULUS III NAME _____

EXAM, FALL 1999 Rec. Instr. _____

Rec. Time _____

TO RECEIVE CREDIT YOU MUST SHOW YOUR WORK.

(25) 1. An object is moving in 3-space according to the parametric equations $x = 2\cos t$, $y = t$, $z = \sin t$. Find, as functions of the time t,

a) position vector $\vec{r} =$

b) velocity vector $\vec{v} =$

c) acceleration vector $\vec{a} =$

d) speed $\dfrac{ds}{dt} =$

e) tangential component of acceleration $a_T =$

f) curvature $\kappa =$

g) normal component of acceleration $a_N =$

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(15) 2. An object is moving in the plane along the curve $y = \dfrac{1}{x}$, $x > 0$ in the direction of increasing x. Its <u>speed</u> <u>is</u> <u>constant</u> at 4 ft/sec.

a) Find a_T and a_N when the object is at $\left(x, \dfrac{1}{x}\right)$.

b) Find the velocity vector and the acceleration vector when the object is at the point $(1, 1)$.

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(25) 3. Let $f(x,y) = y^3 - 3y\cos x$. Find and classify all the critical points of $f(x,y)$ which lie in the strip $-\dfrac{\pi}{4} < x < \dfrac{9\pi}{4}$.

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(15) 4. Use the method of Lagrange multipliers to find the largest value and the smallest value of $f(x, y, z) = z + xy$ on the ellipsoid $x^2 + y^2 + \frac{1}{2}z^2 = 8$.

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(25) 5. Find the equation for each of the following planes:

a) Plane containing the point $(2, -1, 3)$ and perpendicular to the line $x = 1 + 3t$, $y = 4t$, $z = 2 - t$.

b) Plane containing the points $(1, 1, 1)$, $(2, 1, 3)$ and $(1, -1, 2)$.

c) Plane which is tangent to the surface $x^2y + y^2z = z^2x + 5$ at the point $(1, 2, 3)$.

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(10) 6. Let $f(x, y, z) = xy + yz + xz$. Find the instantaneous rate of change of $f(x, y, z)$ per unit distance at the point $(x, y, z) = (1, 2, 3)$ in the direction of the vector $\vec{i} - 4\vec{j} + 2\vec{k}$.

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(10) 7. Calculate the volume of the region in the first octant which is enclosed by the surfaces $y = x^2$, $y = 4$, $x = 0$, $z = 3$, and $y + z = 8$.

(15) 8. A mass distribution occupies the 3-d region which is above the xy-plane and between the spheres $x^2 + y^2 + z^2 = 4$ and $x^2 + y^2 + z^2 = 16$. If the mass density function is $\delta(x, y, z) = (x^2 + y^2)z$ units of mass/unit volume, calculate the total mass.

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(15) 9. The force field $\vec{F} = \vec{i} + (x+y)\vec{j}$ acts on an object as it moves in the plane. Calculate the work done by $\vec{F}$ as the object moves from $(0,0)$ to $(0,\sqrt{2})$ along the following path:

first, along $y = x$ from $(0,0)$ to $(1,1)$,

second, along $x^2 + y^2 = 2$ from $(1,1)$ to $(0,\sqrt{2})$ in the first quadrant.

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(15) 10. Let $f(x,y) = x^2 + 4xy + y^2$. Calculate the value of the outward flux integral $\int_C (\nabla f) \cdot \vec{n}\, ds$ where C is the ellipse $x^2 + 2y^2 = 4$.

(10) 11. Evaluate the surface integral $\int\int_S z\, dS$ where S is the hemisphere $x^2 + y^2 + z^2 = 4$, $z \geq 0$.

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(20) 12. Verify Stokes' theorem by directly calculating both sides of the equation, in the special case where the vector field is $\vec{F} = y\vec{i} + z\vec{k}$ and the surface is $z = 4 - x^2 - y^2$, $z \geq 0$ oriented with its upward normal.