CALCULUS III _____ NAME: _____

EXAM _____ REC. INSTR. _____

Fall 1997 _____

TO RECEIVE CREDIT YOU MUST SHOW YOUR WORK.

(35) 1. A particle is moving in the plane according to $x = 2\cos(t^2)$, $y = 2\sin(t^2)$ where t is the time.

(a) Draw the curve along which the particle is moving.

Find as functions of t,

(b) position vector $\mathbf{r} =$

(c) velocity vector $\mathbf{v} =$

(d) speed $\dfrac{ds}{dt} =$

(e) tangential component of acceleration $a_T =$

(f) normal component of acceleration $a_N =$

(g) unit tangent vector $\mathbf{T} =$

(h) unit normal vector $\mathbf{N} =$

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(20) 2. A particle is moving in 3-space in such a way that its acceleration vector, as a function of time, is given by $\mathbf{a} = (-\sin t)\mathbf{i} + \mathbf{k}$. Suppose you know that at time $t = 0$ its velocity vector is $\mathbf{v}(0) = \mathbf{i} + \mathbf{j}$. Find as functions of t

(a) velocity vector $\mathbf{v} =$

(b) speed $\dfrac{ds}{dt} =$

(c) curvature $\kappa =$

(d) normal component of acceleration $a_N =$

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(20) 3. Given the points in 3-space

$$P(2,3,1), Q(-2,1,0), R(1,6,1)$$

find

(a) the cosine of the angle between $\overrightarrow{PQ}$ and $\overrightarrow{PR}$

(b) area of the triangle with vertices P, Q and R

(c) the equation of the plane containing P, Q and R

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(20) 4. Let $f(x,y) = x^2y - xy^3 + 3x + y^2$.

(a) Find the gradient vector field of f

$$\nabla f =$$

(b) If you are walking along a straight line through the point $(x,y) = (3,1)$ towards the point $(x,y) = (2,2)$, what is the instantaneous rate of change of f per unit distance that you will observe at the point $(3,1)$?

(c) If you are standing at the point $(x,y) = (3,1)$, in which direction should you go to experience the largest possible rate of change in f ?

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(10) 5. Given that $z = f(x, y)$ and that at $(x, y) = (1, 2)$ you know that $\dfrac{\partial f}{\partial x}(1, 2) = 3$ and $\dfrac{\partial f}{\partial y}(1, 2) = -1$. If (r, θ) denote polar coordinates in the xy-plane, use the chain rule to find $\dfrac{\partial z}{\partial \theta}$ and $\dfrac{\partial z}{\partial r}$ at the point $(x, y) = (1, 2)$.

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(20) 6. Let $f(x,y) = x^2(1-e^y) + ye^y$. Find and classify the critical points of $f(x,y)$.

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(15) 7. Use the method of Lagrange multipliers to find the maximum value and the minimum value of $f(x, y) = -x + y^2$ on the ellipse $2x^2 + y^2 = 8$.

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(10) 8. Find the volume of the 3-dimensional region enclosed by the surfaces $y = x$, $x = 2$, $y = 0$, $z = y$ and $z = 2 + x$.

(10) 9. Find the surface area of that part of the paraboloid $z = x^2 + y^2$ which is between the cylinders $x^2 + y^2 = 2$ and $x^2 + y^2 = 9$.

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(10) 10. A mass distribution occupies the region which is inside the sphere $x^2 + y^2 + z^2 = 4$ and above the xy-plane. If the mass density function is $\delta(x, y, z) = z^2\sqrt{x^2 + y^2 + z^2}$ use a triple integral in spherical coordinates to calculate the total mass.

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(10) 11. The force field $\mathbf{F} = y\mathbf{i} - x\mathbf{j}$ acts on an object as it moves in the plane. Calculate the work done by $\mathbf{F}$ as the object moves in the 1st quadrant from $(2, 0)$ to $(0, 2)$ along the circle $x^2 + y^2 = 4$. Assume distance is measured in feet and force in pounds.

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(10) 12. Show that the force field

$$\mathbf{F} = \left(\frac{x}{x^2+y^2}\right)\mathbf{i} + \left(\frac{y}{x^2+y^2}\right)\mathbf{j}$$

is conservative in the region $(x, y) \neq (0, 0)$ by finding a potential function for $\mathbf{F}$. Now use this potential function to find the work done by $\mathbf{F}$ as it acts on an object which moves from $(1, 1)$ to $(4, 3)$ without passing through $(0, 0)$.

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(10) 13. Use Green's theorem to evaluate the line integral

$$\int_C (4x^3y + x^2)dx + (x^4 + y^4 - 3x)dy$$

where C is the entire circle $(x-1)^2 + (y-3)^2 = 25$ directed counterclockwise.