CALCULUS III NAME _____

EXAM FALL 2002 Rec. Instr. _____

Rec. Time _____

TO RECEIVE CREDIT YOU MUST SHOW YOUR WORK.

(20) 1) A particle is moving in 3-space according to the parametric equations $x = t$, $y = t^2$, $z = t^2 + t$ where t is the time. Find, as functions of t,

a) position vector $\vec{r} =$

b) velocity vector $\vec{v} =$

c) acceleration vector $\vec{a} =$

d) speed $=$

e) tangential component of acceleration $a_T =$

f) curvature $=$

g) normal component of acceleration $a_N =$

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(25) 2) Let $f(x, y) = 2xy^2 - x^2y + 4y - 2y^3$. Find all local max points, local min points, and saddle points.

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(15) 3) Use the method of Lagrange multipliers to find the maximum value and the minimum value of $T = x^2 - y + z$ on the ellipsoid $x^2 + 2y^2 + z^2 = 6$.

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(20) 4) Find an equation for each of the following planes:

a) tangent to $x^2y + y^2z^2 + z^4 = 7$ at $(1, 2, 1)$.

b) containing $(1, 3, 2)$ and perpendicular to the line $x = 2t$, $y = 1+t$, $z = 4-3t$.

c) tangent to the parametric surface $x = s + t$, $y = s - t$, $z = 2st$ at the point obtained when $t = 1$ and $s = 2$.

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(15) 5) A solid body occupies the region which is above the paraboloid $z = x^2 + y^2$ and under the plane $z = 10$. Find the total mass if the mass density function is $\delta(x, y, z) = z$.

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(15) 6) Evaluate the surface integral $\int\int_S z^2 dS$ where S is the parametrized surface $x = 4\cos t$, $y = 4\sin t$, $z = s$, $0 \leq t \leq 2\pi$, $0 \leq s \leq 3$.

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(15) 7) Calculate the volume of the 3D region enclosed by the surfaces $y = x^2$, $y = 2x$, $z = y$ and $z = 4 + x$.

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(20) 8) The force field $\vec{F} = (y+1)\vec{i} + (y-x)\vec{j}$ acts as on object as it moves in the plane. Calculate the work done by $\vec{F}$ as the object moves from $(2,0)$ to $(0,4)$ by going first along $x^2 + y^2 = 4$ in the 1st quadrant from $(2,0)$ to $(0,2)$ and then along the y-axis from $(0,2)$ to $(0,4)$.

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(15) 9) <u>Use Green's theorem</u> to evaluate $\int_C (x^2 - y^2)dx + (x^2 + y^2)dy$ where C is the boundary of the square with vertices $(0,0)$, $(1,0)$, $(1,1)$, $(0,1)$ directed counter-clockwise.

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(20) 10) Verify Stokes' theorem by calculation of both sides of the equation where S is the surface $z = 9 - x^2 - y^2$ for $z \geq 0$ oriented with upward pointing normal, and $\vec{F} = xz\vec{i} + (xz + yz)\vec{j} + (x^2 + y^2)\vec{k}$.

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(20) 11) Verify the divergence theorem by calculation of both sides of the equation where the region T is the cube $0 \leq x \leq 2$, $0 \leq y \leq 2$, $0 \leq z \leq 2$, S is its boundary and $\vec{F} = x^2\vec{i} + y\vec{j} + (z+3)\vec{k}$.