CALCULUS III NAME _____

FINAL EXAM Rec. Instr. _____

FALL 2000 Rec. Time _____

TO RECEIVE CREDIT YOU MUST SHOW YOUR WORK.

(15) 1) A particle is moving in 3-space. At a certain instant $t = t_0$ seconds we know that the position vector is $\vec{r}(t_0) = 2\vec{i} - \vec{j} + \vec{k}$, the velocity vector is $\vec{v}(t_0) = \vec{i} + \vec{j} + 3\vec{k}$ and the acceleration vector is $\vec{a}(t_0) = \vec{i} + 2\vec{j} - 2\vec{k}$. **At this instant** $t = t_0$ determine

a) the point at which the particle is located;

b) the speed of the particle;

c) the unit tangent vector;

d) the tangential component of acceleration;

e) the normal component of acceleration;

f) the curvature.

g) Is the object speeding up or slowing down? Why?

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(15) 2) an object is moving in 3-space in such a way that its acceleration vector as a function of time is $\vec{a} = \vec{j} + (\cos t)\vec{k}$. Suppose that at time $t = 0$ you know that its velocity vector is $\vec{v}(0) = \vec{i}$ and its position vector is $\vec{r}(0) = \vec{k}$.

a) Find the velocity vector and the position vector as functions of time t.

b) Give the parametric equations for the motion.

c) Find the curvature at time $t = 2\pi$.

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(15) 3) A quantity Q depends upon x and y according to $Q = \dfrac{e^{xy}}{x^2 + y^2}$. Both x and y are changing with time t and suppose at a certain instant we know that $x = 1$, $y = 2$, $\dfrac{dx}{dt} = -2$ and $\dfrac{dy}{dt} = 6$. Use the chain rule to find $\dfrac{dQ}{dt}$ at this instant.

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(15) 4) Use the method of Lagrange multipliers to find the maximum value and the minimum value of $f(x, y) = y^2 - 2y - x^2$ on the circle $x^2 + y^2 = 4$.

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(20) 5) Given the three points $P(1,0,1)$, $Q(2,0,0)$, and $R(-1,2,2)$

a) Find the area of the triangle having P, Q and R as vertices.

b) Find the angle at the vertex P of the triangle having P, Q and R as vertices.

c) Find the equation of the plane containing the points P, Q and R.

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(15) 6) A mass distribution occupies the region which is between the spheres $x^2+y^2+z^2 = 1$ and $x^2+y^2+z^2 = 4$. If the mass density function is $\delta(x, y, z) = z^4$ units of mass/unit volume, find the total mass.

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(15) 7) Find the volume of the 3-dimensional region enclosed by the surfaces $y = x^2$, $y = 4$, $z = y$ and $z = 10 - y$.

(15) 8) Calculate the area of that part of the surface $z = y^2$ which is above the region of the xy-plane bounded by $y = x$, $x = 0$ and $y = 1$.

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(15) 9) Evaluate the surface integral $\int\int_S x dS$ where S is the parametrized surface

$$x = s\cos t, \quad y = s\sin t, \quad z = t, \quad 0 \le s \le 4, \quad 0 \le t \le \frac{\pi}{4}.$$

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(15) 10) The force field $\vec{F} = y^2\vec{i} + xy\vec{j}$ acts on an object as it moves in the plane. Suppose the object is moved from $(0,0)$ to $(3,6)$ by first going along $y = x^2$ from $(0,0)$ to $(2,4)$ and then along $y = 2x$ from $(2,4)$ to $(3,6)$. Calculate the work done by $\vec{F}$.

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(20) 11) Let S be the closed surface, consisting of two pieces, which bounds the 3-dimensional region which is under $z = 4 - x^2 - y^2$ and above the xy-plane. Suppose $\vec{F} = x\vec{i} + y\vec{j} + e^z\vec{k}$. Verify the divergence theorem in this case by calculating both sides of the equation and seeing that they are equal.

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(20) 12) Verify Stokes' theorem by directly calculating both sides of the equation where the vector field is $\vec{F} = y\vec{i} - x\vec{j} + z\vec{k}$ and the surface is $x^2 + y^2 + z^2 = 4$, $z \geq 0$.