NAME ______________________ Rec. Instr. ______________________
Signature ______________________ Rec. Time ______________________

CALCULUS 2
EXAM III
April 20, 1999

Show all work for full credit. No calculators, books or notes are allowed. The point value of each problem is given in the margin.

problem	1	2	3	4	5	6	7	8	total
points									
points possible	6	10	10	24	20	9	9	12	100

(6) 1. Use your knowledge of the MacLaurin series for $\sin x$ to get (just by substitution) the MacLaurin series for $\sin(x^3)$.

(10) 2. Find the area of one leaf of the rose $r = 2\sin 3\theta$.

(10) 3. Find the area of the region inside both of the circles $r = 4\cos\theta$ and $r = 4\sin\theta$.

(24) 4. In parts (a) through (d) below, determine whether the given *sequence* $\{a_n\}_{n=1}^{\infty}$ converges or if it diverges. If it converges, determine the limit of the sequence. If it diverges, explain why.

(a) $a_n = (-1)^n \dfrac{1}{n}$

(b) $a_n = \sqrt[n]{2^{n+1}}$

(c) $a_n = \dfrac{\ln(2n)}{\ln(3n)}$

(d) $a_n = 1 + \dfrac{1}{3} + \ldots + \dfrac{1}{3^n} = \sum_{k=0}^{n} \left(\dfrac{1}{3}\right)^k.$

(20) 5. For each of the following *series* $\sum a_n$, determine whether the series converges or diverges. Justify your answers, as precisely as you can. (If the series converges, you need <u>not</u> find its limit.)

(a) $\sum\limits_{n=1}^{\infty} (.999)^n$

(b) $\sum\limits_{n=0}^{\infty} \dfrac{2^{n+3}}{3^n}$

(c) $\sum\limits_{n=1}^{\infty} \left(-\dfrac{\pi}{3}\right)^n$

(d) $\sum\limits_{n=1}^{\infty} \dfrac{n^2-5n-1}{5n^2+10n+30}$

(9) 6. Find the third Taylor polynomial $P_3(x)$ for $f(x) = \cos x$, at $a = \pi/4$, and write down an expression for the corresponding error term.

(9) 7. Find the MacLaurin series for $\cosh x = \dfrac{e^x + e^{-x}}{2}$.

(12) 8(a) Express the rectangular equation $x^2 - y^2 = 25$ in terms of polar coordinates.

(b) Express the polar equation $r^3 = \cos\theta$ in terms of rectangular coordinates.

(c) Find an equation for the line through the origin of slope -1, in terms of polar coordinates.