NAME _____ Rec. Instr._____ Signature
_____ Rec. Time_____

ANALYTIC GEOMETRY AND CALCULUS II
EXAM III
April 21, 1998

Show all work for full credit. You may use a calculator, but no books or notes.

	1	2	3	4	5	total
points per page						

Some formulas for conic sections:

$$a = \frac{c}{e}; \qquad a^2 = b^2 + c^2; \qquad c^2 = a^2 + b^2; \qquad \text{directrix } \frac{c}{e^2}; \qquad x^2 = 4py; \qquad \frac{x^2}{a^2} \pm \frac{y^2}{b^2} = 1$$

(10) 1. Graph the conic section $x^2 + 2x - y^2 + 2y + 4 = 0\,.$

(10) 2. Show that the curve $r = \dfrac{\cos\theta}{\sin^2(\theta)}$ is a parabola, and find its focus.

(10) 3. If the asymptotes of a hyperbola are $y = \pm x$, find its eccentricity.

(10) 4. Find an equation of the ellipse with eccentricity $.6$ and foci $(1, 2)$ and $(1, 4)$.

(10) 5. Sketch a graph of $r = 1 - 4\sin^2(\theta)$. Label the angles where $r = 0$.

(10) 6. Find the area inside $r^2 = 4\sin(2\theta)$.

(10) 7. If the series converges, compute its sum. Otherwise, explain why it diverges.

(a) $\displaystyle\sum_{n=1}^{\infty} \left(\frac{e}{\pi}\right)^n$

(b) $\displaystyle\sum_{n=1}^{\infty} \frac{2^n - 1}{2^n}$

(10) 8. Find the degree three Taylor polynomial $P_3(x)$ at $a = 0$ for the function $f(x) = x\cos(2x)$.

(10) 9. Show that the sequence $a_n = \left(1 - \frac{3}{n}\right)^n$ converges.

(10) 10. Peter, Paul, and Mary take turns tossing a coin until one of them wins by tossing the first "head". Calculate for each person the probability that they win the game.