CALCULUS II – EXAM 3 – FALL 2000
NO CALCULATORS

NAME ______________________________
Rec. Instr. ______________________________
Rec. Time. ______________________________

1. (26pts). Integrate:

a. (13pts) $\displaystyle\int \frac{dt}{1-t^2}$

b. (13pts) $\displaystyle\lim_{N\to\infty}\int_1^N xe^{-x}dx$

2.(26pts) A cyclist is pedaling uphill, from right to left, at a constant speed of 50 cm/sec, on a 30° incline. A light is attached to a spoke of the front wheel, 25 cm from the center. At time $t = 0$, the center of the front wheel is at $(0, 0)$ and the light is at $(0, 25)$. The wheels have a radius of 50 cm.

a. (6pts) Find the components of the velocity vector for the center of the front wheel and deduce equations for the motion of the center (x_c, y_c).

b. (6pts) Let θ be the usual angle (increases counterclockwise about the positive x-axis). Find $d\theta/dt$ and then $\theta(t)$.

c. (14pts) Write parametric equations for the motion of the light.

3.(26pts) The logarithmic spiral has parametric equations

$$\begin{aligned} x(t) &= e^{-t}\cos t \\ y(t) &= e^{-t}\sin t \end{aligned}$$

a. (6pts) Compute $(x(0), y(0))$ and $(x(\pi), y(\pi))$.

b. (6pts) Compute $(dx/dt, dy/dt)$.

c. (14pts) Compute the length of the spiral from $t = 0$ to $t = \pi$. (Recall arc-length $= \sqrt{(dx/dt)^2 + (dy/dt)^2}$).

4. (26pts) A vat contains 100 gallons of water. At time $t = 0$, 5 kg of salt are added in the vat, so the initial concentration is $C_0 = 5/100$ kg/gal. Assume also that fresh water enters the vat at 2 gal/min and the well-stirred mixture leaves the vat at the same rate.

a. (6pts) If $Q(t)$ is the amount of salt at time t, then it must satisfy the equation $dQ/dt = -2(Q/100)$. Solve this equation for Q and deduce the concentration $C(t) = Q(t)/100$ of salt in the vat, at time t.

b. (10pts) Suppose now that 5 kg of salt are added at time $t = 10$, $t = 20$, and so on, while the rate of water going in and out is left unchanged. What is the concentration of salt right after 30 minutes?

c. (10pts) The concentration of salt in the vat will tend to a stable concentration c_∞. Compute it.