CALCULUS II – EXAM 2 – SPRING 2000
NO CALCULATORS

NAME ______________________________
Rec. Instr. ______________________________
Rec. Time. ______________________________

1.(20pts) Integrate (show all the steps):

a. $\displaystyle\int \frac{x+10}{2x^2+5x-3}$

b. $\displaystyle\int_0^{\infty} xe^{-x}dx$

2.(20pts) Integrate (show all the steps):

a. $\int x^2 \cos(2x)dx$

b. $\int \frac{dx}{\sqrt{x^2 - 2x + 3}}$. **Use** $\int \frac{du}{u^2 + a^2} = \ln|u + \sqrt{u^2 + a^2}| + C.$

3. (20pts) A metal rod of length 1 has density $\rho(x) = x\ln(1/x)$, where x is the distance from, say, the left end-point. Assume that the rod is lying between 0 and 1.

a. (5pts) Find $\lim_{x\to 0}\rho(x)$ and $\lim_{x\to 1}\rho(x)$ (Use L'Hôpital when needed.)

b. (5pts) Find the critical point of $\rho(x)$ and graph $y = \rho(x)$ for $0 \le x \le 1$ (Use $e^{-1} = .36$).

c. (10pts) Compute the center of gravity of this rod

$$\bar{x} = \int_0^1 x\rho(x)dx \Big/ \int_0^1 \rho(x)dx =$$

4. (20pts) **Water is flowing at 3 m^3/min in one end and at the same rate out the other end of a vat containing 100 m^3 of water. Let $y(t)$ be the amount (in grams) of impurity in the vat of water at time t min. The water flowing in contains 0.1 gm of impurity per cubic meter. Set up a differential equation for $y(t)$ and integrate it. Then graph $y(t)$ in each of the following cases: the initial concentration is (a) 0.05 gm/m^3, (b) 0.20 gm/m^3, (c) 0.10 gm/m^3.**

5. (20pts) A beetle population on an island obeys the logistic equation. The maximum beetle population that the island can support is 100 million. At present there are 5 million beetles on the island, and last year at this time there were 2 million beetles. Find a formula for the beetle population at time t. How long will it be before the population reaches 95 million?

6. (10 Extra pts) It is an interesting fact that the inflection point in the solution of a logistic equation $\frac{dy}{dt} = ky(M-y)$ occurs at $y = M/2$. Verify this by differentiating both sides of the logistic equation (the logistic equation will also help you determine for which values of y you might have $y' = 0$.) What can you deduce if the initial value y_0 is greater then $M/2$?