CALCULUS II – EXAM 1 – SPRING 2001
NO CALCULATORS

NAME ____________________________
Rec. Instr. ____________________________
Rec. Time. ____________________________

1.(16pts) Differentiate the following functions:

a. $f(x) = x \ln \sqrt{x}$

b. $g(t) = e^{2 - \ln t}$

(16pts) Integrate:

a. $\int \frac{1}{e^{3t}} dt =$

c. $\int_1^2 \frac{dx}{2x - 5} =$

2. (28pts) Let $f(x) = 10x^{1/2} \ln \dfrac{200}{x}$, for $x > 0$.

a. (4pts) Find the following limits:

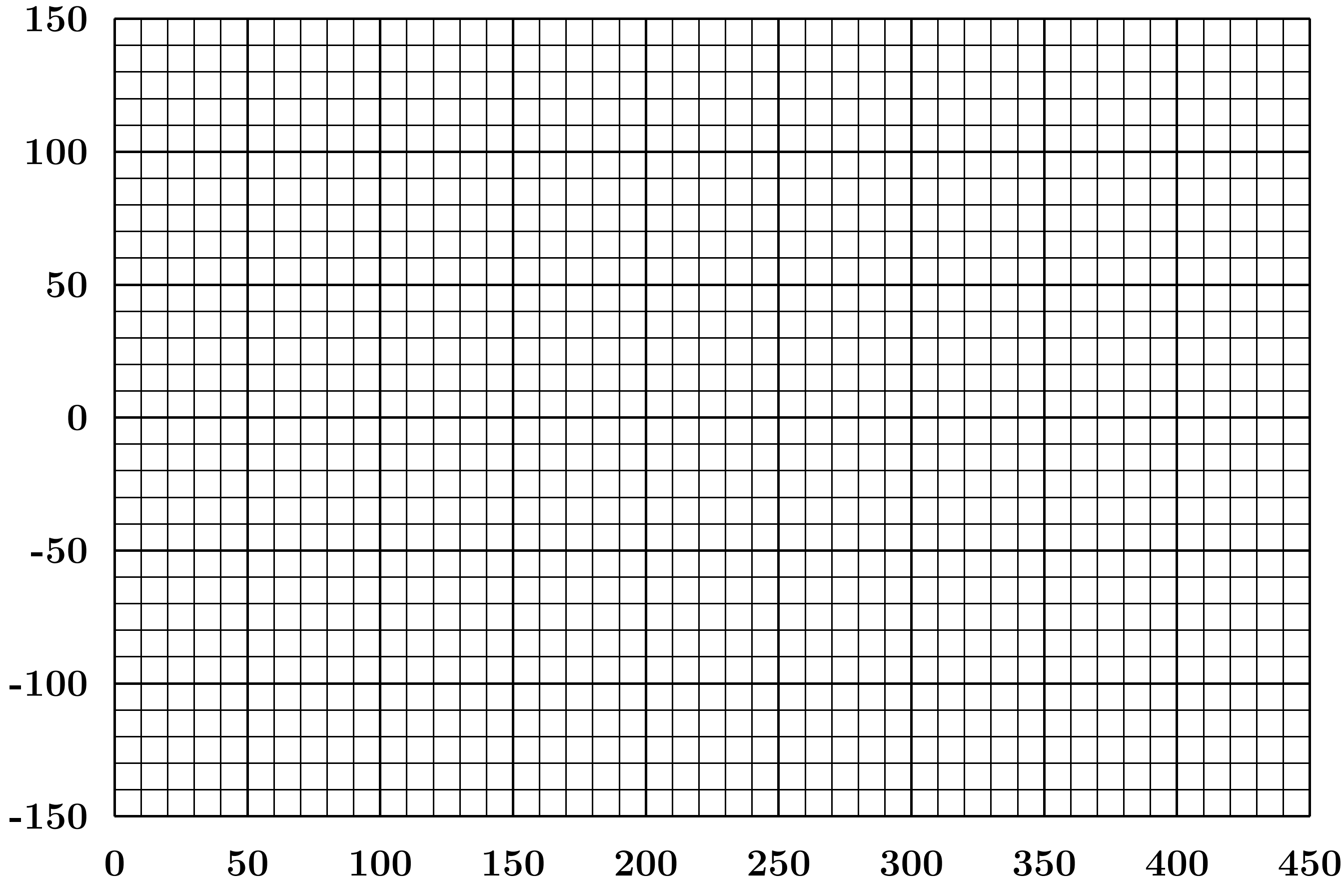
$\lim_{x \to 0^+} f(x) =$

$\lim_{x \to +\infty} f(x) =$

b. (6pts) $f'(x) = 0$ has one solution. Find it.

c. (6pts) $f''(x) = 0$ has one solution. Find it

d. (12pts) Evaluate $f(x)$ at the critical point and at the inflection point, evaluate $f'(x)$ at 0 (limit) and at the inflection point, and find where f crosses the x-axis. Then graph $y = f(x)$, labeling all the relevant features. Use the following approximate values: $e^{-1} = .36$, $e^{-2} = .13$, $\sqrt{2} = 1.4$, $1/\sqrt{2} = .7$.



3. **(20pts) The Tuttle Creek Reservoir has a volume of $10km^3$ (I'm inventing here). Water comes in at a rate of $0.1km^3/day$ and leaves at the same rate. Suppose that one day a plant starts dumping a certain pollutant into the water upstream from the reservoir, so that the water coming in has now a concentration of $1kg/km^3$ of pollutant. Set up a differential equation for the amount $Q(t)$ of pollutants in the reservoir at time t and solve it step by step. We assume the pollutants to be well-stirred at all times.**

4. (15pts) A population grows at a rate proportional to its present amount. Suppose that at time $t = 0$ there are $P(0) = 1000$ individuals and the rate of change is $P'(0) = 20$ individuals/day. How long will it take for the population to double ($\ln 2 = 0.69$)?

5. (15pts) In Problem 3. the amount of pollutant in the lake reaches the unbearable level of $5kg/km^3$ and the state of Kansas orders the plant to close down and decides to drain the reservoir. So fresh water now comes in at the same rate as before, but the outgoing flow is increased to its maximum rate of $-0.2km^3/day$. Find a formula for the rate of change of the volume $V(t)$ of water in the lake at time t and integrate it to find a formula for $V(t)$. Use $V(t)$ to set up a differential equation for the amount $Q(t)$ of pollutants in the reservoir at time t. Then solve it step by step.