CALCULUS II – EXAM 1 – SPRING 2000
NO CALCULATORS

NAME _____
Rec. Instr. _____
Rec. Time. _____

1.(20pts) Differentiate the following functions:

a. $f(x) = \sqrt{x}\ln x$

b. $y = (\ln x)^x$

c. $g(t) = e^{3+\ln t}$

d. $f(x) = \arctan(e^x)$

2.(20pts) Integrate:

a. $\int_2^3 \frac{dx}{x \ln^2 x} =$

b. $\int_0^{2\pi} \frac{\sin\theta}{2 - \cos\theta} d\theta =$

c. $\int_0^1 \frac{1}{10^t} dt =$

d. $\int \frac{x^4}{1 + x^{10}} dx =$

3. (24pts) Let $f(x) = \dfrac{x^3}{4 \cdot 10^4} \ln \dfrac{x}{3 \cdot 10^2}$, for $x > 0$.

a. (6pts) Find the following limits:

$\lim_{x \to 0^+} f(x) =$

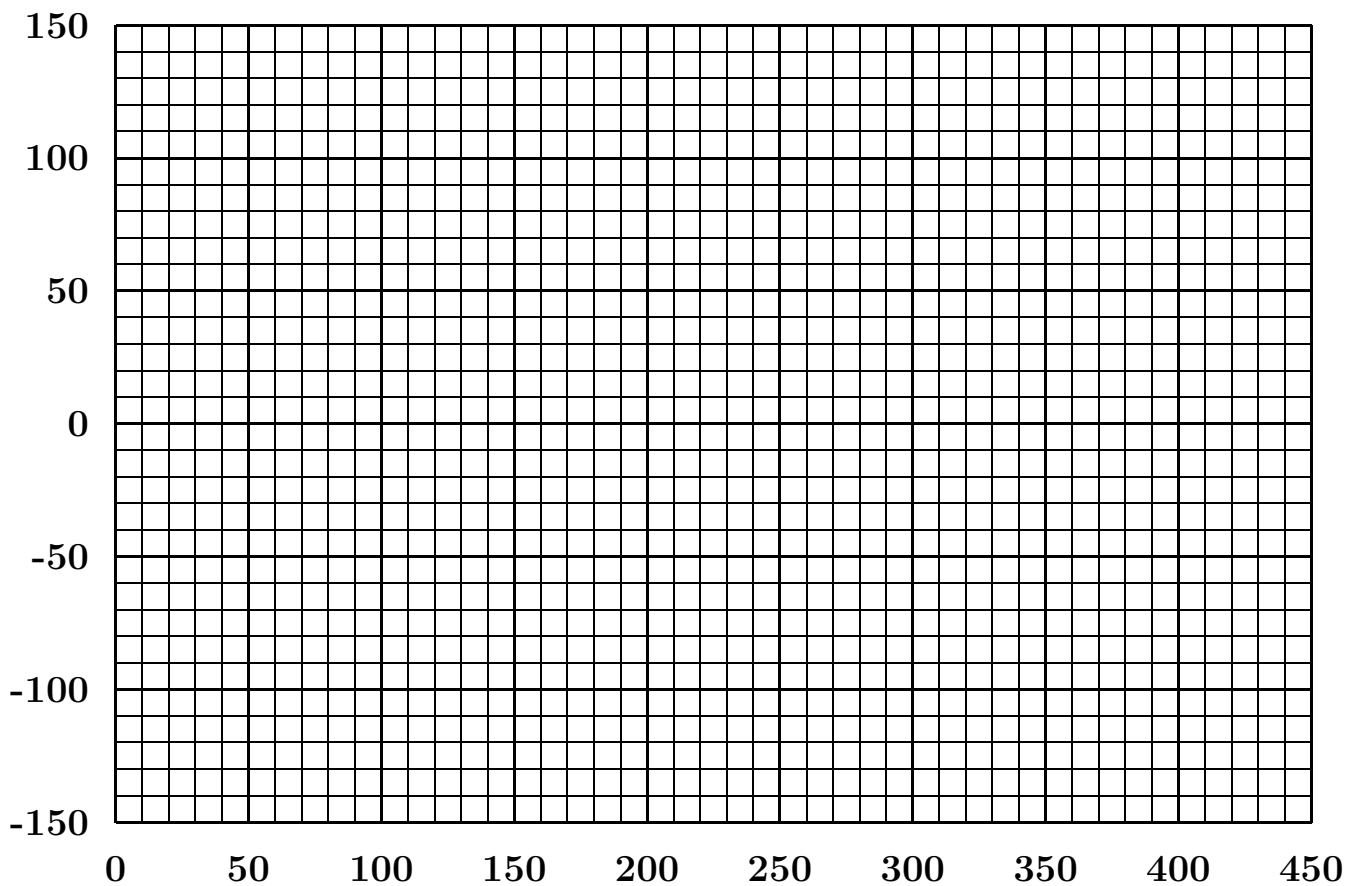
$\lim_{x \to +\infty} f(x) =$

b. (5pts) $f'(x) = 0$ has one solution. Find it.

c. (5pts) $f''(x) = 0$ has one solution. Find it

d. (2pts) How does $f(x)$ behave for x small? In particular, what is the sign of $f(x)$ for x small?

e. (6pts) Graph $y = f(x)$, labeling the extremum and the inflection point. (Hint: to evaluate $f(x)$ at the critical point and at the inflection point use the following approximate values:
$e^{-1} = .36$, $e^{-1/3} = .72$, $e^{-5/6} = .43$, $e^{-5/2} = .08$)



4. (16pts) Word Problems.

a.(10pts) Suppose that interest on money in the bank accumulates at an annual rate of 7% compounded continuously. How much money should be invested today so that 20 years from now it will be worth \$ 20000? (Hint: use $e^{1.4} = 4$)

b.(6pts) One of the guiding principles of most sports is "keep your eye on the ball". In baseball, a batter stands 2 feet from home plate as a pitch is thrown with a velocity of 130 ft/sec (about 99 mph). At what rate does the batter's angle of gaze change when the ball crosses home plate? (Aside: humans can accurately track objects only at the rate of 3 radians/second.)

5. (20pts) Suppose that a room containing 1000 cubic feet of air is originally free of carbon monoxide. Beginning at time $t = 0$ cigarette smoke containing 4 percent carbon monoxide is introduced into the room at a rate of 0.1 ft^3/min, and the well-circulated mixture is allowed to leave the room at the same rate.

a.(10pts) Find an expression for the amount $y(t)$ of carbon monoxide (in cubic feet) in the room at time t, by solving a differential equation of the form $dy/dt = C_{in} \cdot r - C_{out} \cdot r$, where C stands for concentration and r stands for rate (you'll need to figure out C_{in}, C_{out}, and r from the word problem first).

b.(10pts) Extended exposure to a carbon monoxide concentration as low as 0.012% is harmful to the human body. Find the time at which this concentration is reached. (Use the fact that $\ln(1 + x)$ is approximately equal to x when x is small.)

6. (10 Extra pts) You drop a rock (with zero initial velocity) from a bridge. Take $g = 10\text{m/sec}^2$. Because of air resistance (which we assume to be proportional to the velocity $v(t)$), the rock would eventually approach a downward terminal velocity of 100 m/sec if it fell indefinitely through the air. However, it does not fall indefinitely: after exactly 10 seconds it hits the water. How fast is the rock falling when it hits the water? (use $e^{-1} = .36$)