Math 221 - Review for Final Exam

- Exponential and Logarithmic functions: definitions, properties, derivative and integration.
  a) Simplify $\log_3 \frac{1}{9}$ b) $y = \log_4(\cos x)$ find y'
- Implicite differentiation; derivatives of inverse functions
  $xe^y = y$
- Logarithmic differentiation: $y = \sqrt{\frac{x-1}{x+1}}$
- Natural growth and decay, half-life.
  $P_0 = 2$ mil, rate of growth 20% find doubling time.
- Linear first order differential equations: separation of variables.
  Solve $P' = 20P - 10, P(0) = 10$.
- Inverse trigonometric functions; derivatives
  Use implicite differentiation to find $\sin^{-1}(x)$
- Indeterminate forms and L'Hopital's rule
  a) Find $\lim_{x\to\infty}(1 + \frac{1}{x})^x$ b) $\lim_{x\to 0} \frac{e^x - 1}{\sin x}$
- Hyperbolic functions and inverse hyperbolic functions
  Use the definition of $\sinh x$ to solve for x in $2 = \sinh x$
- Integration methods: substitutions, trigonometric substitutions and integration by parts. Integrate:
  a) $\int \frac{e^{-\frac{1}{x}}}{x^2}\, dx$ b) $\int \frac{1}{\sqrt{1-x^2}}\, dx$ c) $\int \sin x \cos^2 x\, dx$
  d) $\int \sqrt{x} \ln x\, dx$ e) $\int$ arcsin $x\, dx$ f) $\int x^2 e^x dx$ g) $\int \sin x\, e^x\, dx$

<u>Integration</u>

9.5 - Rational functions and Partial Fraction Decomposition

9.6 - Trigonometric substitutions $a^2 - u^2$ $\sin x$, $u^2 - a^2$ $\frac{1}{\sin x}$, $a^2 + u^2$ $\tan x$

9.7 - Integrals containing quadratic polynomials (complete the square and use a trigonometric (or hyperbolic) substitution (or use the formula giving some inverse trigonometric function).

9.8 Improper integrals: defined as limits of definite integrals. The limit point is +/- infinity or a discontinuity of the integrand.

<u>Conic Sections</u>

Locus: The set of points P such that PF = e PL (F focus, L directrix, e eccentricity).

10.4 - Parabola; $y^2 = 4px$, V(0,0), Focus(p, 0), Directrix x=-p

10.5 - Ellipse; $\frac{x^2}{a^2} + \frac{y^2}{b^2}$ Center(0,0), Foci(+/-c, 0), $c^2 = a^2 - b^2$ $0 < b < a$.

10.6 - Hyperbola $\frac{x^2}{a^2} - \frac{y^2}{b^2}$ Center(0,0), Foci(+/-c, 0), $c^2 = a^2 + b^2$ $0 < b < a$, asymptotes $y = \frac{b}{a}x$

10.7 - Equations for Rotation of Axes and Classification of Conics (discriminant $B^2 - 4AC$)

10.2 - Polar Coordinates $x = r\cos\theta (or \cos\theta = \frac{adj.}{hyp.})$ $y = r\sin\theta (or \sin\theta = \frac{opp.}{hyp.})$

Polar curves $<->$ Cartesian curves, intersecting polar curves.

10.3 - Area computations in polar coordinates. $dA = \frac{1}{2}r^2 d\theta$. Area between two curves.

<u>Infinite Series</u>

11.2 - Infinite Sequences

Limit laws (+/-, ./:), Squize Law, Bounded monotonic Sequence.

Recursive relations and sequences.

Problems

1) $\displaystyle\int \frac{x^3+x+1}{x(x^2+1)}\,dx$

2) $\displaystyle\int \frac{1}{4x^2+4x+5}\,dx$

3) $\displaystyle\int x\sqrt{1-x^2}\,dx$

4) $\displaystyle\int \frac{1}{(x^2+1)^{3/2}}\,dx$

5) Find the area under the curve $y=\frac{1}{x^2}$ from 1 to infinity.

6) Determine if the improper integral for the curve $y=\frac{1}{\sqrt[3]{x}}$ from -1 to 3 is convergent. Find the volume of the solid obtained by rotating the curve about the x-axis.

7) Find the equation to describe all the points P(x,y) which are equidistant from (3,2) and (7,4).

8) Draw the graph of the polar equation $r=2+3\cos\theta$.

9) Find the area outside of r=2 and inside $r=4\cos\theta$.

10) Sketch the following conic section (label the foci, vertices, directrices, axes, center): $x^2-4y-4x=0$

11) Sketch the graph of $\dfrac{(y-2)^2}{4}-\dfrac{(x+1)^2}{9}=1$

12) Determine if the following sequences converge: a) $a_n=\frac{3^n}{n!}$, b) $a_n=\frac{n^2}{e^n}$ c) $a_n=(\frac{e}{10})^n$ d) $a_n=\frac{n^e}{e^n}$

13) Determine if the sequence $a_{n+1}=\sqrt{a_n+2}, a_1=2$ converges and if so, compute its limit.

- Geometric series, telescopic sums, p-series, harmonic series.
  Determine whether the infinite series converges or diverges. If it converges find its sum:
  a) $\sum_{n=0}^{\infty} 42^{-n}$ b) $\sum_{n=0}^{\infty}\frac{3}{n^2+2n}$

- Nth term test for divergence, integral test and remainder estimate, comparison test, limit comparison test.
  Determine whether the infinite series converges or diverges. If it converges find its sum:
  a) $\sum_{n=0}^{\infty}\frac{\sqrt[3]{n}}{n^4+5}$ b) $\sum_{n=0}^{\infty}\frac{\sin(\frac{\pi}{2})}{7n}$ c) $\sum_{n=0}^{\infty}\frac{n!}{2^n}$

- Taylor's formula, series and polynomial.
  Find the 4th degree Taylor formula for $f(x)=\cos x$ at 0.

- Power series, radius and interval of convergence, term-wise differentiation and integration, algebra of power series.
  1) Write the power series representation for $\frac{1}{1+x}$, then term-wise integrate to find the Taylor series of $\ln x$ at 0. Use Taylor's formula to estimate the remainder and finally compute $\ln(1.1)$ accurate to three decimal places.
  2) Find the radius and interval of convergence for $\sum_{n=0}^{\infty}\frac{x^n}{n}$.