Math 221 - Review for Test #2

Integration

9.5 - Rational functions and Partial Fraction Decomposition

9.6 - Trigonometric substitutions

$a^2 - u^2 \quad \sin x, \quad u^2 - a^2 \quad \frac{1}{\sin x}, \quad a^2 + u^2 \quad \tan x$

9.7 - Integrals containing quadratic polynomials (complete the square and use a trigonometric (or hyperbolic) substitution (or use the formula giving some inverse trigonometric function).

9.8 Improper integrals: defined as limits of definite integrals. The limit point is +/- infinity or a discontinuity of the integrand.

Conic Sections

Locus: The set of points P such that PF = e PL (F focus, L directrix, e eccentricity).

10.4 - Parabola; if the vertex at the origin: $y^2 = 4px$, Focus(p, 0), Directrix x=-p

10.5 - Ellipse; if Center(0,0): $\frac{x^2}{a^2} + \frac{y^2}{b^2}$ Foci(+/-c, 0), $c^2 = a^2 - b^2 \quad 0 < b < a$ (major semiaxes is a)

10.6 - Hyperbola (two branches) Center(0,0): $\frac{x^2}{a^2} - \frac{y^2}{b^2}$ Foci(+/-c, 0), $c^2 = a^2 + b^2$ 0¡b¡a (major semiaxes is a, transverse semiaxes b) asymptotes $y = \frac{b}{a}x$

10.7 - Equations for Rotation of Axes and Classification of Conics (discriminant $B^2 - 4AC$)

10.2 - Polar Coordinates $x = r\cos\theta (or \cos\theta = \frac{adj.}{hyp.}) \quad y = r\sin\theta (or \sin\theta = \frac{opp.}{hyp.})$

Polar curves $<->$ Cartesian curves, intersecting polar curves.

10.3 - Area computations in polar coordinates.

$dA = \frac{1}{2}r^2 d\theta$. Area between two curves r_1 and r_2 : $dA = dA_1 - dA_2$.

Infinite Series

11.2 - Infinite Sequences

Limit laws (+/-, ./:), Squize Law, Bounded monotonic Sequence.

Recursive relations and sequences.

Problems

Integrate:

1) $\displaystyle\int \frac{x^3 + x + 1}{x(x^2+1)}\, dx$

2) $\displaystyle\int \frac{1}{4x^2 + 4x + 5}\, dx$

3) $\displaystyle\int x\sqrt{1 - x^2}\, dx$

4) $\displaystyle\int \frac{1}{(x^2+1)^{3/2}}\, dx$

5) Find the area under the curve $y = \frac{1}{x^2}$ from 1 to infinity.

6) Determine if the improper integral for the curve $y = \frac{1}{\sqrt[3]{x}}$ from -1 to 3 is convergent. Find the volume of the solid obtained by rotating the curve about the x-axis.

7) Find the equation to describe all the points P(x,y) which are equidistant from (3,2) and (7,4).

8) Draw the graph of the polar equation $r = 2 + 3\cos\theta$.

9) Find the area outside of r=2 and inside $r = 4\cos\theta$.

10) Sketch the following conic section (label the foci, vertices, directrices, axes, center): $x^2 - 4y - 4x = 0$

11) Sketch the graph of $\dfrac{(y-2)^2}{4} - \dfrac{(x+1)^2}{9} = 1$

12) Determine if the following sequences converge: a) $a_n = \frac{3^n}{n!}$, b) $a_n = \frac{n^2}{e^n}$ c) $a_n = (\frac{e}{10})^n$ d) $a_n = \frac{n^e}{e^n}$

13) Determine if the sequence $a_{n+1} = \sqrt{a_n + 2}, a_1 = 2$ converges and if so, compute its limit.