NAME ____________________ Rec. Instr. ____________________

Signature ____________________ Rec. Time ____________________

CALCULUS 2
FINAL EXAM
May 12, 1999

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points							
page	7	8	9	10	11	12	total
points							

Show all work for full credit. No calculators, books or notes are allowed. The point value of each problem is given in the margin.

(18) 1. Find the derivatives of the following functions.

(a) $f(x) = e^{\sin(\ln x)}$

(b) $g(x) = \ln(\sqrt{e^x + e^{-x}}) - \ln 2$

(c) $$\begin{aligned} h(x) &= 1 + \frac{x}{3} + \frac{x}{5} + \ldots + \frac{x^n}{2n+1} + \ldots \\ &= \sum_{n=0}^{\infty} \frac{1}{2n+1} x^n \end{aligned}$$

(Express your answer in part (c) as a power series.)

(12) 2. The half-life of radioactive lavernium is 100 hours. Starting with a certain quantity of lavernium, we find that 12 kilos of lavernium is left after 350 hours. What was the initial amount of lavernium?

(12) 3.Find the area of the cardioid $r = 2 - 2\sin\theta$.

(15) 4. Evaluate the following limits.

(a) $\lim_{x \to 0} \frac{\sin x^2}{x}$

(b) $\lim_{x \to \infty} \frac{x^{10} + x^9 + x^8 + \ldots + x + 1}{x^{10} + 2x^9 + 3x^8 + \ldots + 10x + 11}$

(c) $\lim_{n \to \infty} x^{(1/x^2)}$

(10) 5. Use the integral test to decide whether $\sum_{n=1}^{\infty} ne^{-n^2}$ converges.

(40) 6. Here are four integrals to work out. (The last is on the next page.)

(a) $\int_0^{\pi/8} \tan^2 2x \, dx$

(b) $\int xe^x \, dx$

(c) $\int \frac{dx}{x(x-2)}$

(d) $\int \sqrt{4 - 9x^2}\, dx$

(15) 7. Find the volume of the solid of revolution obtained by revolving the graph of $y = \dfrac{1}{\sqrt{x}}$, from $x = 1$ to $x = e$, around the x-axis.

(15) 8. Determine whether the given <u>sequence</u> $\{a_n\}_{n=1}^{\infty}$ converges or diverges. If it converges, find the limit of the sequence. If it diverges, explain why.

(a) $a_n = e^{-n}$

(b) $a_n = (-1)^n \dfrac{n}{n+1}$

(c) $$\begin{aligned} a_n &= 1 - \frac{1}{4} + \frac{1}{16} - \frac{1}{64} + \ldots + (-1)^n \left(\frac{1}{4}\right)^n \\ &= \sum_{k=0}^{n} (-1)^k \left(\frac{1}{4}\right)^k \end{aligned}$$

(10) 9. Find an anti-derivative of $\cos x^3$ (expressed as a power series).

(20) 10. Determine whether the given series $\sum a_n$ converges or diverges. Justify your answer.

(a) $\sum\limits_{n=31}^{\infty} (-1)^n \left(\frac{1}{4}\right)^n$

(b) $\sum\limits_{n=1}^{\infty} \frac{n^2}{3n+1}$

(c) $\sum\limits_{n=0}^{\infty} \left(\frac{n}{n+1}\right)^n$

(d) $\sum\limits_{n=1}^{\infty} \frac{\sin\left(\frac{1}{n}\right)}{n}$

(21) 11. Find the interval of convergence for each of the following power series.

(a) $\sum\limits_{n=1}^{\infty} \dfrac{x^n}{\sqrt{n}}$

(b) $\sum\limits_{n=1}^{\infty} (-1)^n \dfrac{x^n}{(2n)!}$

(c) $\sum\limits_{n=1}^{\infty} n!\, x^n$

(12) 12.(a) Write out the first four terms in the binomial series expansion of $\dfrac{1}{\sqrt[3]{8+x^3}}$

(b) Find the radius of convergence of the above series.