NAME _____ Rec. Instr._____ Signature
_____ Rec. Time_____

ANALYTIC GEOMETRY AND CALCULUS II
FINAL EXAM
May 13, 1998

Show all work for full credit. You may use a calculator, but no books or notes.

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Points										

(10) 1. Find an equation for the tangent line to the curve $y = \left(1 + \frac{1}{x}\right)^x$ at the point $x = 1$.

(12) 2. Find the area under the curve $y = (lnx)^2$ between $x = 1$ and $x = e$.

(12) 3. A chain 100 feet long and weighing 15 lbs/ft is hanging from the roof of a tall building. How much work is done in lifting the chain to the roof?

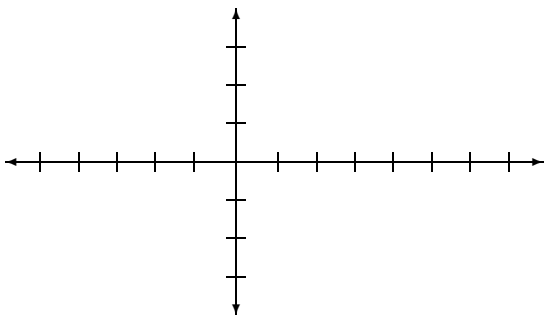
(12) 4. Find the center of an ellipse which is the set of points $P = (x, y)$ such that the distance from P to the focus $F = (2, 5)$ is equal to $\frac{2}{3}$ of the distance from P to the directrix $y = 0$ (the x-axis).

(12) 5. Find the coordinates of the vertices of the hyperbola with asymptotes $y = x + 3$ and $y = -x + 1$, and with one focus at $F = (3, 2)$.

(12) 6. Evaluate the limit.

$$\lim_{x \to 0} \left(\frac{1}{x} - \frac{1}{\ln(x+1)} \right)$$

(21) 7. (a) Graph the curve $r = -2 + 4\cos(\theta)$. List the angles where $r = 0$.



(b) Give integrals (DO <u>NOT</u> EVALUATE) representing the area <u>between</u> the two loops of $r = -2 + 4\cos(\theta)$.

(24) 8. Evaluate the integrals.

(a) $\displaystyle\int \frac{x^3}{x^2-4}\,dx$

(b) $\displaystyle\int \tan^3(x)\sec(x)\,dx$

(10) 9. Evaluate the integral.

$$\int \frac{e^x dx}{1 + e^{2x}}$$

(15) 10. Approximate $\int_0^{\frac{1}{2}} e^{-2x^3} dx$ using the Taylor series for e^{-2x^3}. Use enough terms to get an accuracy of two decimal places, and estimate the error in your approximation of the definite integral.

(10) 11. Determine whether the series converges or diverges. Explain.

$$\sum_{n=1}^{\infty} \frac{n}{2n-1}$$

(15) 12. Find the interval of convergence of the power series

$$\sum_{n=1}^{\infty} \frac{2^n (x-1)^n}{n}.$$

(20) 13. Determine whether the series converge or diverge. Explain.

(a) $\displaystyle\sum_{n=1}^{\infty} \frac{n}{\sqrt{n^4+1}}$

(b) $\displaystyle\sum_{n=2}^{\infty} \frac{1}{n(\ln(n))^2}$

(15) 14. A small but undetected and continual leak at a nuclear power plant releases 2 milligrams of radioactive cesium each year. The half-life of cesium is 30 years.

(a) Write a differential equation describing the change in the amount of cesium in the neighborhood of the power plant.

(b) If the leak began at time $t = 0$, write the amount of cesium as a function of time (solve the differential equation and find all constants).