CALCULUS II – FINAL – SPRING 2001
NO CALCULATORS

NAME _____

Rec. Instr. _____

Rec. Time. _____

1. (24pts). Integrate

a. (12pts) $\int \frac{1}{x^2 - 2x - 2} dx$

b. (12pts) $\int_0^1 x \arctan(x) dx$

2. (30pts) Let $f(x) = 10x^{1/2}e^{-\frac{x}{200}}$, for $x > 0$.

a. (8pts) $f'(x) = 0$ has one solution. Find it and evaluate f there. (Use the Taylor expansion to two decimal places to estimate $e^{-0.5}$.)

b. (8pts) $f''(x) = 0$ has one solution. Find it and evaluate f there. ($\sqrt{2} = 1.4$, $\sqrt{240} = 15.5$, $e^{-1.2} = 0.3$)

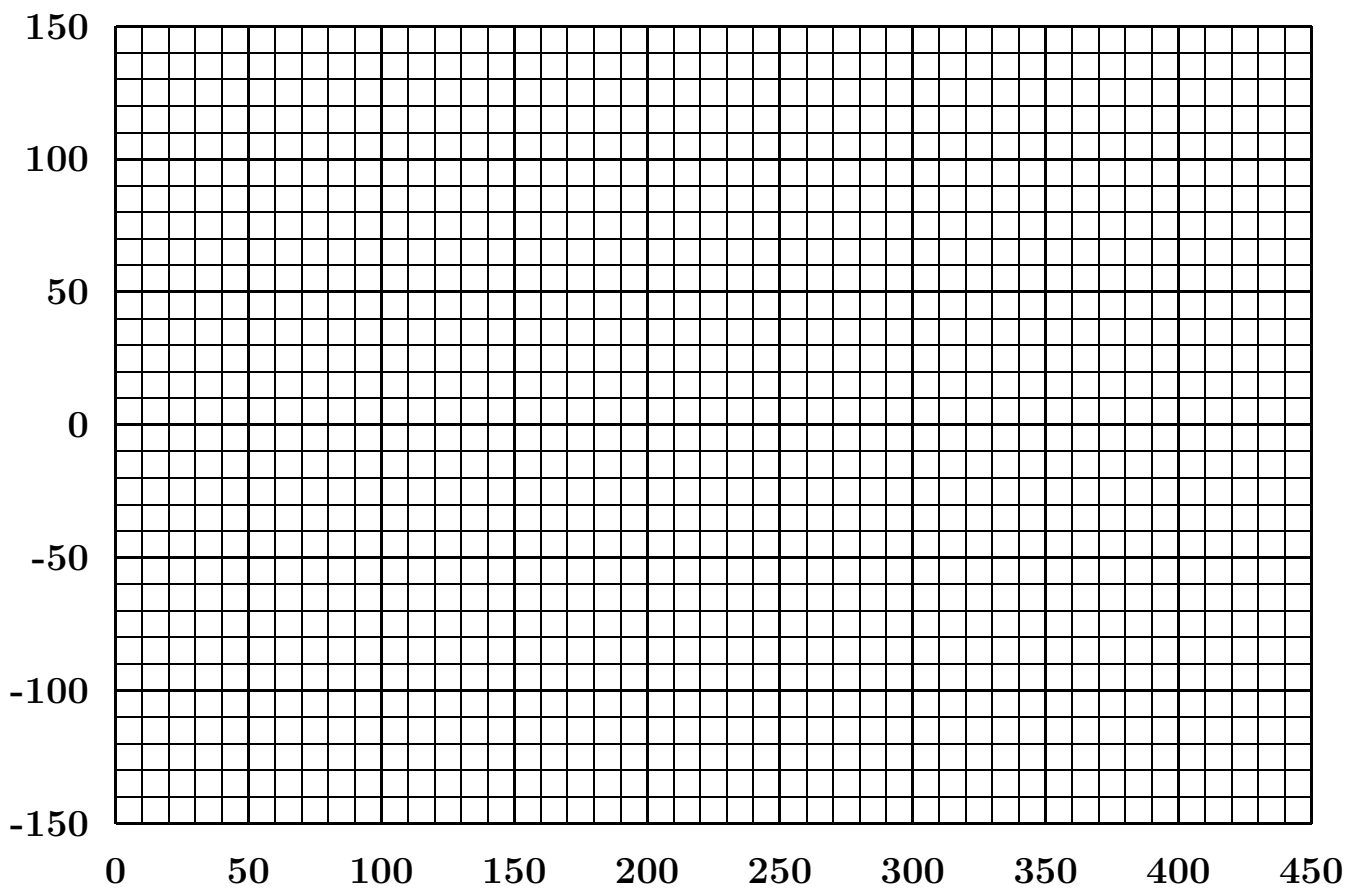
a. (6pts) Find the following limits:

$$\lim_{x \to 0^+} f(x) =$$

$$\lim_{x \to +\infty} f(x) =$$

$$\lim_{x \to 0^+} f'(x) =$$

d. (8pts) Graph $y = f(x)$, labeling all the interesting points.



3. (30pts) Matt, the cheerleader, is twirling a baton of length 2 ft about the center-point. Assume that his fingers touch the center of the baton at a height of 6 ft above the ground.

a.(4pts) Write equations for the motion of the tip (X_T, Y_T) of the baton, assuming that at time $t = 0$ the baton is horizontal, the tip we are considering is the right-most tip, and the twirling is counter-clockwise, at 3 revolutions per second.

b.(8pts) After a few complete revolutions, Matt throws the center (X_C, Y_C) of the baton at a 45° angle above the horizontal, with an initial velocity v_0 ft/sec. Write equations for (X_T, Y_T) in terms of v_0. (You can restart the time here.)

c. (8pts)Suppose that another cheerleader, Jen, is standing on the shoulder of Tom, 4 ft to the right of Matt, with her hands stretched out to reach a height of 9 ft. Assuming that Matt throws the center as in part b., what should v_0 be in order for the center to reach Jen's hands?

d.(6pts) What are the components of the velocity vector of the center, its magnitude, and its angle above the horizontal, when the baton reaches Jen's hands?

e.(4pts) What is the inclination of the baton when it reaches Jen's hands?

4. (26pts) a.(13pts) Expand $f(x) = \dfrac{1}{2-x}$ in a power series about 0 and find the interval of x's for which this expansion converges.

b.(13pts) Compute the third degree Taylor polynomial of $f(x) = \dfrac{e^{-x}}{1-x}$ at 0.