CALCULUS II – FINAL – SPRING 2000
NO CALCULATORS

NAME _____
Rec. Instr. _____
Rec. Time. _____

1.(16pts) Integrate (show all the steps):

a.(10pts) $\displaystyle\int \frac{e^x}{e^{2x}+3e^x+3}dx =$

b.(6pts) $\displaystyle\int_2^\infty \frac{1}{x\ln x}dx =$

2.(20pts) A positive function $f(x)$, for $x \geq 0$, is called a probability density function if $\int_0^\infty f(x)dx = 1$.
a. Find the value of the constant C which makes $f(x) = Cxe^{-2x}$ into a probability density function.

b. Find limits, extrema and inflection points, and then graph the function $f(x)$ found in a. for $x \geq 0$.

c. Compute the mean $\int_0^\infty xf(x)dx$.

3. (20pts)

a. At what rate, annual and compounded continuously (a.c.c.), should you invest 5,000$ today so as to have 20,000$ in 8 years? (Use $\ln 2 = .69$).

b. You find a good deal and invest your 5,000$ at 20% (a.c.c.). At the same time, a friend of yours invests 10,000$ in a different bank, but only gets 15% (a.c.c.). How long will it take for you and your friend to have the same amount of money? (Use $\ln 2 = .69$).

4. (20pts) You are riding on a ferris wheel, 20 ft in radius. Suppose that at t=0 you are at the bottom of the wheel, at (0,0), and that the wheel rotates clockwise, taking 2 min. to complete one loop.
a. Write parametric equations for your motion.

b. Compute the velocity vector v=(dx/dt,dy/dt) after 15 seconds.

c. At t=1 min. you drop your soda which therefore falls to the ground. Neglecting air resistance, write equations for the coordinates of the soda.

5. (20pts) A vat contains $100m^3$ of water with an initial concentration of 2% of salt. Water is pumped in at one end and out at another end at the same variable rate of $\cos(\pi t/100)m^3/sec$, for $0 \leq t \leq 50$. The incoming water contains 1% of salt, while the water that is leaving the vat is assumed to be well-stirred. Write a differential equation for the amount of salt $y(t)$ at time t, and integrate it.

6. (14pts) Expand $f(x) = x^2 e^x$ in a power series about the origin, and compute its radius of convergence.