Name (Please Print) _____ Rec. Instr. _____

Your Signature _____ Class Time _____

ANALYTIC GEOMETRY AND CALCULUS II
Final Exam
Fall 1999

Be neat, clear and complete. Show <u>all</u> work. No notes, books, reference material or calculators are allowed. No credit will be given for answers without supporting work.

1. Differentiate:

(8) a) $D_x(\sin^{-1}(3^x))$

(8) b) $D_x(\cosh^2 x + \sinh^2 x)$

(8) c) $D_x(\ln(\tan^{-1} x))$

2. Integrate: If an integral is improper, so note and treat it as such, determining its value if it converges or determining that it diverges.

(9) a) $\displaystyle\int_{x=0}^{\infty} \frac{dx}{4+x^2}$

(9) b) $\displaystyle\int \frac{dx}{-x^2 - 2x + 3}$

(9) c) $\displaystyle\int \csc^3 x \cot^3 x \, dx$

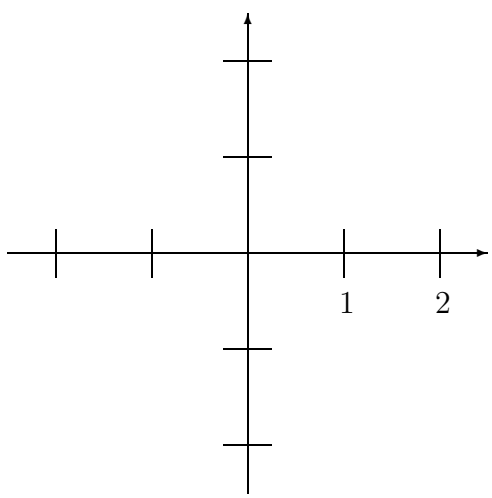
(9) d) $\displaystyle\int_1^e x^2 \ln x \, dx$

(9) e) $\displaystyle\int \frac{x}{\sqrt{4x^2 - 9}} \, dx$

(9) f) $\displaystyle\int \frac{3x^2 + 3x + 4}{(x+2)(x^2+1)} \, dx$

(8) 3. Find the limit: $\lim_{x \to 0+} (\sin x)^{\sin x}$

(12) 4. Graph the polar equation $r = \sin 2\theta$ and find the area inside it.



θ	$r = \sin 2\theta$
0	
$\pi/4$	
$\pi/2$	
$3\pi/4$	
π	
$5\pi/4$	
$3\pi/2$	
$7\pi/4$	
2π	

(8) 5. If $P(x)$ grows exponentially, $P(1) = 10$ and $P(2) = 15$, find $P(5)$. Leave your answer in terms of exponentials and/or logarithms.

6. Investigate the convergence of each of the following series. Tell whether the given series converges conditionally, converges absolutely or diverges. You must justify your answer with a relevant test.

(9) a) $\displaystyle\sum_{n=1}^{\infty} \frac{e^{2n}}{n!}$

(9) b) $\displaystyle\sum_{n=1}^{\infty} \left(\frac{\ln n}{\sqrt{n}}\right)^n$

(9) c) $\displaystyle\sum \frac{(-1)^{n+1}5^n}{3^n + 4^n}$

(9) d) $\displaystyle\sum_{n=2}^{\infty} \frac{1}{n(\ln n)^{3/2}}$

(9) e) $\displaystyle\sum_{n=2}^{\infty} \frac{(-1)^{n+1}}{\ln n}$

(10) 7. Determine the interval convergence of the power series: $\displaystyle\sum_{n=1}^{\infty} \frac{(-1)^{n+1}(x+2)^n}{n}$.

Be sure to investigate the behavior of the series at the endpoints of the interval of convergence (if there are any).

(10) 8. Write the first four non-zero terms of the MacLaurin series for $e^x \cos x$.

(10) 9. Write the first four terms of the binomial series for $\sqrt[3]{1+x^4}$, $|x| < 1$.

10. If $f(x) = \displaystyle\sum_{n=1}^{\infty} \frac{x^n}{n^3}$ for $-1 < x < 1$,

(10) a) write a power series representation of $f'(x)$, $-1 < x < 1$.

(10) b) write a power series representation for $\displaystyle\int_0^x f(t)\,dt$, $-1 < x < 1$.