NAME _____ Rec. Instr._____

Signature _____ Rec. Time_____

CALCULUS II
FINAL EXAM
December 16, 1998

Show all work for full credit. No calculators, books or notes are allowed. The point value of each problem is given in the margin.

Problem	Points	Points possible	Problem	Points	Points possible
1		24	7		10
2		12	8		35
3		20	9		16
4		12	10		16
5		10	11		15
6		18	Total Score		200

SHOW YOUR WORK CLEARLY.

$$\cos^2 x = \frac{1}{2}(1 + \cos 2x)$$

$$\sin^2 x = \frac{1}{2}(1 - \cos 2x)$$

(24) 1. Find the derivatives. (Do not try to simplify your answers!)

(a) $e^{(e^x)}$

(b) $\tan^{-1}(\ln(2x))$

(c) $x^{\sin x}$

(12) 2. Evaluate the limits.

(a) $\lim_{x \to 0} \dfrac{e^x - x - 1}{2x}$

(b) $\lim_{x \to \infty} x\left(e^{1/x} - 1\right)$

(20) 3. Evaluate the integrals. (Don't simplify your answers.)

(a) $\displaystyle\int \sin^2(4x)\,dx$

(b) $\displaystyle\int x \ln x\,dx$

(c) $\displaystyle\int \frac{dx}{x(2x+1)}$

(d) $\displaystyle\int \frac{dx}{\sqrt{9-4x^2}}$

(12) 4. Let b be a positive real number. Find the volume obtained by revolving the region between the graph of $y = e^{-x}$, the x-axis, and the vertical lines $x = 0$ and $x = b$, around the x-axis.

(10) 5. Immediately after an accident at a nuclear power plant, the radioactivity level is 10 times the "safe" limit. After 6 months the radioactivity drops to 8 times the safe limit. Assuming that the level of radioactivity decays exponentially, we have

$$L(t) = C\,e^{-kt}$$

where $L(t)$ is the level of radioactivity at time t, and where C and k are constants. Find the constant k. [I realize that you don't have a calculator, but you don't need it for this problem. The answer can be expressed using $\ln$.]

(18) 6. Determine whether the given sequence $\{a_n\}_{n=1}^{\infty}$ converges or diverges. If it converges, find the limit of the sequence.

(a) $a_n = \dfrac{2^n + 3^n}{5^n}$

(b) $a_n = (-1)^n \dfrac{1}{n!}$

(c) $a_n = 1 + \frac{1}{2} + \frac{1}{4} + \cdots + \left(\frac{1}{2}\right)^n$

(10) 7. Find the third Taylor polynomial $P_3(x)$ for e^x, at $a = 1$.

(35) 8. Determine whether the given series converges or diverges. Provide some justification for your answer.

(a) $\displaystyle\sum_{n=1}^{\infty} \frac{n}{n^3+1}$

(b) $\displaystyle\sum_{n=1}^{\infty} (.998)^n$

(c) $\displaystyle\sum_{n=1}^{\infty} \frac{n!}{2^n}$

(d) $\displaystyle\sum_{n=1}^{\infty} (-1)^n \frac{n^2}{n^3+1}$

(e) $\displaystyle\sum_{n=1}^{\infty} \left(\sin\frac{1}{n}\right)^n$

(16) 9. (a) Use the integral test to decide whether $\displaystyle\sum_{n=1}^{\infty} \frac{1}{n\ln(n)}$ converges.

(<u>Hint</u>: The relevant anti-derivative can be found easily by making a straightforward substitution.)

(b) Same question for $\displaystyle\sum_{n=1}^{\infty} \frac{1}{n(\ln(n))^2}$.

(16) 10. Find the radius of convergence for each of the following power series.

(a) $\displaystyle\sum_{n=0}^{\infty} \frac{x^{2n}}{n!}$

(b) $\displaystyle\sum_{n=1}^{\infty} (-1)^n x^{2n}$

(15) 11. (a) Find the MacLaurin series for $\dfrac{1}{1+x^2}$ by making a substitution in the series for $\dfrac{1}{1-x}$.

(b) What is the radius of convergence of the series that you found in part (a)?

(c) Find an anti-derivative of the MacLaurin series for $\dfrac{1}{1+x^2}$.