Name (Please Print) ______________________ Rec. Instr. ______________________

Your Signature ______________________ Class Time ______________________

ANALYTIC GEOMETRY AND CALCULUS II

Final Exam

December 17, 1997

Be neat, clear and complete. Show all work. No notes, books, reference material or calculators are allowed. No credit will be given for answers without supporting work.

1. Test each of the following numerical series to determine whether it converges conditionally, converges absolutely, or diverges. **MAKE SURE TO STATE THE TEST YOU ARE USING, AND HOW IT APPLIES.**

(10) (a) $\displaystyle\sum_{n=2}^{\infty} \frac{1}{n(\ln n)^2}$

(10) (b) $\displaystyle\sum_{n=2}^{\infty} \frac{(-1)^n n}{\ln n}$

(10) (c) $\displaystyle\sum_{n=0}^{\infty} \frac{2}{\sqrt[3]{n^2+1}}$

(10) (d) $\displaystyle\sum_{n=2}^{\infty} \frac{(-1)^n}{\sqrt{n}-1}$

(12) 2. Find the interval of convergence of the power series

$$\sum_{n=0}^{\infty} \frac{(-1)^n (x-2)^n}{n+1}$$

Be sure to check the convergence at the endpoints (if there are any).

(10) 3. (a) If the Maclaurin series for f is $\displaystyle\sum_{n=1}^{\infty} \frac{2^n x^n}{n!}$ write the Maclaurin series for $f'(x)$.

(12) (b) Write the Maclaurin series for $\sin(2x) + \cos(3x)$.

4. Differentiate:

(9) (a) $D_x \ln|1 - x^2|$

(9) (b) $D_x (\sin x)^{\sin x}$

5. Integrate:

(10) (a) $\displaystyle\int \tan\theta \sec^3\theta d\theta$

(10) (b) $\displaystyle\int x \sinh x dx$

(10) (c) $\displaystyle\int \frac{-x^2+2x+1}{(x^2+1)(x+1)}\, dx$

(10) (d) $\displaystyle\int \frac{x^3}{\sqrt{4-x^2}}\, dx$

(12) 6. Find the area inside the rose $r = 2\cos 3\theta$

(11) 7. (a) Find the foci of the conic

$$4x^2 + 16x - 25y^2 = 84$$

(11) (b) Write an equation of the hyperbola with foci $(2,4)$ and $(8,4)$ and vertices $(0,4)$ and $(10,4)$.

(10) 8. Find the limit: $\lim_{x \to 0} (1 + \sin x)^{1/x}$

(12) 9. A population grows exponentially. The initial $(t = 0)$ population is 100 and when $t = 5$ hours, the population is 500. How long does it take the population to double to 200? Leave your answer in terms of logarithms.

(12) 10. Find the volume of revolution when the curve

$$f(x) = \frac{1}{\sqrt{1+x^2}}, 1 \leq x < \infty$$

is rotated about the x-axis.

(10) EXTRA CREDIT

Check the series $\displaystyle\sum_{n=0}^{\infty} \frac{n!}{n^n}$ for convergence.