CALCULUS II – FINAL – FALL 2000
NO CALCULATORS

NAME _____

Rec. Instr. _____

Rec. Time. _____

1. (27pts). Integrate: a. $\int \frac{1+t}{4t^2+1}dt$

b. $\int_1^\infty \frac{\ln x}{x^2}dx$

c. Does $\sum_{n=1}^{\infty} \frac{1}{n^3}$ **converge or diverge (justify)?**

2.(26pts) A fish is dropped out of an airplane flying at 80 ft/sec, and 300 ft above the surface of a lake. Let $(V_x(t), V_y(t)) = (dx/dt, dy/dt)$ be the velocity vector of the fish at time t. Write parametric equations $(x(t), y(t))$ for the flight of the fish assuming that the acceleration $(dV_x/dt, dV_y/dt)$ consists of the gravity vector $(0, -32\ ft/sec^2)$, plus air resistance which contributes a deceleration equal to $-\vec{V}(t) = (-V_x(t), -V_y(t))$. (Assume that at $t = 0$ the fish is at the point $(0, 300)$ and the airplane travels from left to right.)

3.(26pts) a. Write parametric equations for the motion of a point $P = (x(t), y(t))$, starting at $(3, 6)$, and rotating counterclockwise about the point $(3, 3)$, with a constant angular velocity of 3 revolutions per second.

b. Reset the clock at $t = 0$ and suppose that the center $(x_c(t), y_c(t))$ which is initially at $(3, 3)$ starts a circular motion of its own, clockwise about the origin, with a constant angular acceleration of 3 rad/sec^2, while P continues its circular motion about (x_c, y_c) with the radius and angular velocity specified in part a. Write the new equations of motion for P.

4. (27pts) a. Use the geometric series $\sum_{n=0}^{\infty} x^n = \frac{1}{1-x}$ to expand $f(x) = \frac{x^2}{(1-x)^2}$ at 0. What is the radius of convergence?

b. Write the third degree Taylor polynomial for $F(x) = e^{2x}\cos(\pi x)$ at $x = 0$.

c. Find the set of x's for which the power series $\sum_{n=0}^{\infty} (x-1)^n$ converges.