Name _____ Instructor _____

Analytic Geometry and Calculus II
Math 221 - Final Exam A
August 1, 1997

You must show all relevant work to receive full credit. The point value of each problem is shown in the left hand margin.

(10) 1. Compute y' using logarithmic differentiation.

$$y = \frac{\sqrt{x^2+1}}{x(x-1)}$$

(10) 2. Compute the following limit.

$$\lim_{x\to\infty}\left(1+\frac{1}{e^x}\right)^x$$

(20) 3. Integrate.

(a) $\displaystyle\int \frac{1}{(x^2+1)^2}\, dx$

(b) $\displaystyle\int \tan^{-1} x\, dx$

(20) 4. Integrate.

(a) $\displaystyle\int \frac{1}{x^3 - 6x^2 + 9x}\, dx$

(b) $\displaystyle\int \frac{1}{x^2 + 10x + 26}\, dx$

(10) 5. Evaluate the limit.

$$\lim_{x \to 0} (\cos x)^{\frac{1}{x^2}}$$

(10) 6. Sketch the graph of the following equation. Be sure to indicate vertices and foci.

$$4x^2 + 9y^2 = 24x$$

(20) 7. Solve the initial value differential equation.

$$\frac{dy}{dx} = 5y + 2 \quad , \qquad y(0) = 2$$

(10) 8. Find the area of the region inside of $r = \cos 3\theta$.

(20) 9. Determine whether the infinite series converges or diverges. If it converges, find its sum. If it diverges, explain why.

(a) $\pi + \frac{\pi}{2} + \frac{\pi}{4} + \frac{\pi}{8} + \frac{\pi}{16} + \ldots$

(b) $\sum_{n=3}^{\infty} \frac{1}{n^2 - 3n + 2}$

10. Determine whether these infinite series converge absolutely, converge conditionally or diverge. Completely explain and justify all conclusions.

(10) (a) $\sum_{n=1}^{\infty} \frac{1}{\sqrt{n^2+4}}$

(10) (b) $\sum_{n=1}^{\infty} \frac{\sqrt{n}}{n^5+2}$

(10) (c) $\sum_{n=1}^{\infty} \frac{\cos(\pi n)}{2n}$

(10) (d) $\sum_{n=1}^{\infty} \frac{n!}{n^n}$

(10) (e) $\sum_{n=1}^{\infty} \frac{(-1)^n n}{3^n} 2^n$

(10) 11. Find the interval of convergence of the power series.

$$\sum_{n=1}^{\infty} \frac{(3x+1)^n}{2\sqrt{n}}$$

(10) 12. Use infinite series to approximate $ln(1.3)$ accurate to three decimal places.

Hint: $ln(1+x) = x - \frac{x^2}{2} + \frac{x^3}{3} - \frac{x^4}{4} \ldots$