Have a great summer.

CALCULUS I

FINAL EXAM

SPRING 1998

NAME _____

Rec. Instr. _____

Rec. Time _____

Find the limit or state that it doesn't exist.

(3 pts) 1. a) $\lim_{x \to 4} \dfrac{\sqrt{x} - 2}{2x - 8}$

(2 pts) b) $\lim_{x \to \pi} \dfrac{x}{\sin x}$

Find but do not simplify.

(4 pts) 2. a) $D_u\left(\sqrt{u^2+1}\right)$

(4 pts) b) $\dot{z}$ if $4z^2+t^3-zt=0$

(4 pts) c) $\dfrac{dy}{dx}$ if $y=\left(\cos\frac{11}{\pi}\right)x^3+\tan\dfrac{\pi}{11}+\sec(2x)$

(4 pts) d) $\dfrac{dy}{dx}$ if $y=\left(3x\sqrt{x^2+1}+2\right)^{-2}$

(2 pts) e) $f'(z)$ if $f(z)=\dfrac{\tan(z^2)}{z^2+1}$

Find.

(4 pts) 3. a) $\displaystyle\int \frac{2x^3 - 3\sqrt{7x}}{x^3}\,dx$

(4 pts) b) $\displaystyle\int \csc^2(5x)\,dx$

(4 pts) c) $\displaystyle\int \frac{x^2}{\sqrt[3]{x^3+1}}\,dx$

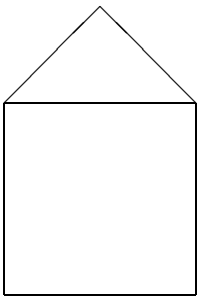
(4 pts) d) $\displaystyle\int \frac{x}{\sqrt[3]{x+1}}\,dx$

(4 pts) e) $\displaystyle\int (\tan x)^5 \sec^2 x\,dx$

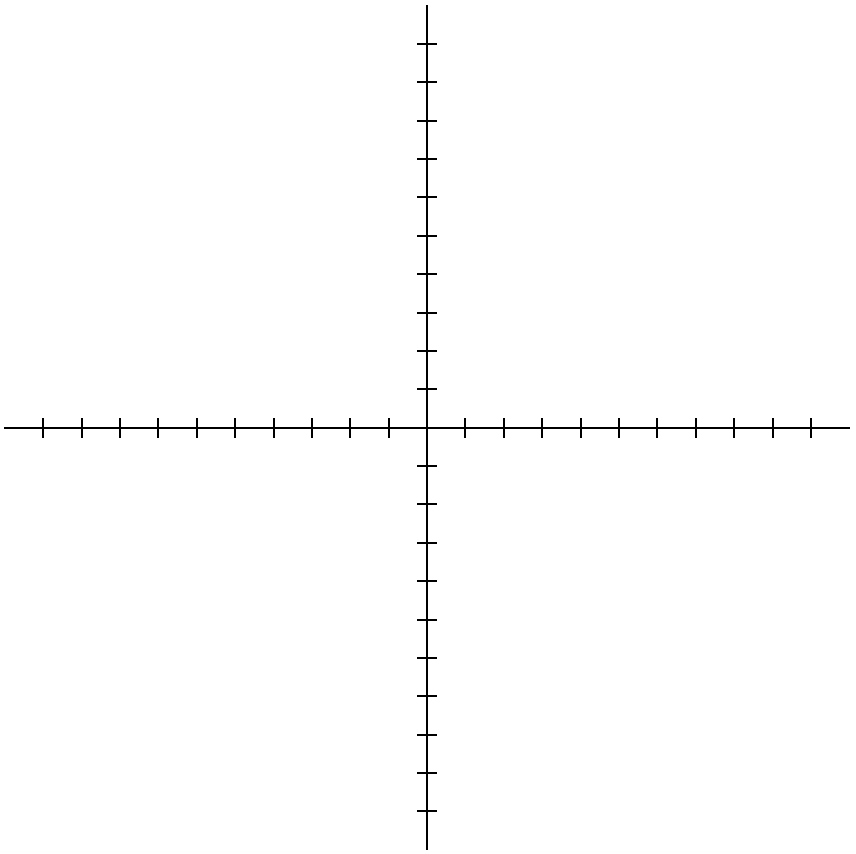
(8 pts) 4. a) A rectangular plate is 2 cm by 5 cm. As the plate is heated, the 2 cm side grows at .02 cm/sec. The 5 cm side grows at .05 cm/sec. How fast is the area of the plate growing at this instant in cm^2/sec ?

(2 pts) b) Over 10 seconds, how much will the area increase?

(10 pts) 5. A $45° - 45° - 90°$ triangle is on top of a rectangle. The total outside perimeter is 6 inches. What is the best upper bound on the area of this region? That is, find the maximal area of the region.



(7 pts) 6. Graph $y = x^4 - 4x^3 + 5$.



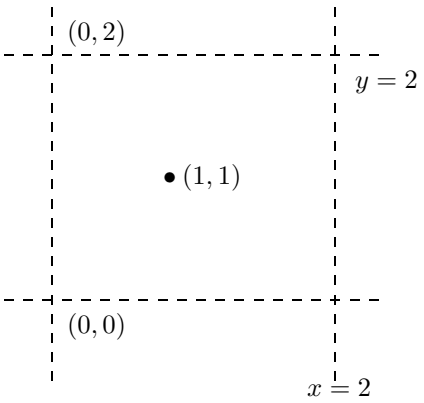
(1 pt) a) For which x-values is the above graph both decreasing and concave up?

(5 pts) 7. Find the area bounded by $x = y^2 - 4$ and $x = 4 - y^2$. **Work out all of the integrals so that your final answer is a number.**

(5 pts) 8. The side view of a door stop is described by $y = 4 - x^2$ and $y = 0$.

Vertical slices perpendicular to the side are right triangles with base length four times the heights. **Write an integral that represents the volume of this region, but do NOT evaluate the integral.**

(3 pts) 9. a) Write a formula representing the volume generated by rotating the region bounded by $y = x^2 - 2x + 2$, $y = x^2$ and $x = 0$ around the axis $x = 2$. **Do NOT evaluate the integral.**



Answer _____

(3 pts) b) Write a formula for the length of the arc $\{(x, y)|y = x^2 - 2x + 2,\ x \in [0, 1]\}$. **Do NOT evaluate the integral.**

Answer _____

(3 pts) c) Write a formula for the area generated by rotating the arc $\{(x, y)|y = x^2 - 2x + 2,\ x \in [0, 1]\}$. **Do NOT evaluate the integral.**

Answer _____

10. Let $f(x) = \begin{cases} x & \text{if } x < 2 \\ 3 - x & \text{if } x \geq 2 \end{cases}$. Hint: Draw a graph.

(1 pt) a) Find $\lim_{x \to 2^-} f(x)$.

(1 pt) b) Find $f(2)$.

(1 pt) c) Find $\int_0^3 f(t)\,dt$.

(1 pt) 11. a) What should you do to the equation $\int_1^x f(t)\,dt = x^3$ to eliminate the integral?

(2 pts) b) Find $f(x)$ given that $f(1) = 4$ and $\int_1^x f(x)\,dt = x^3$.

(1 pt) 12. a) Complete the equation: $\frac{d}{dx}(x \sin x) =$

(1 pt) b) Integrate both sides of the equation in part a) to derive a formula for $\int x \cos x\,dx$.