NAME ______________________________ Rec. Instr., Time __________________

CALCULUS I - ALTERNATE FINAL EXAM
December 17, 2004

<u>Show all work</u> for full credit. The point value of each problem is given in the left-hand margin.

(21) 1. Evaluate the following limits.

a) $\lim_{x\to 5} \dfrac{25x - x^3}{x+5} =$

b) $\lim_{h\to 0} \dfrac{h}{\tan h^2} =$

c) $\lim_{x\to -\infty} \dfrac{\sqrt{x^2+10}}{2x} =$

(12) 2. Find the equation of the tangent line to the curve $x^2 + y^4 = 5$ at the point $(2, 1)$.

(16) 3. Compute the following derivatives. **DO NOT SIMPLIFY**

a) $f'(t)$ where $f(t) = \sqrt{(3t+1)^3 - 15}$.

b) $\dfrac{d}{dx}\,\dfrac{\sin^2(x)}{x-3} =$

(12) 4. Let $f(x) = x(2+x)^{1/3}$. Answer the following questions given that

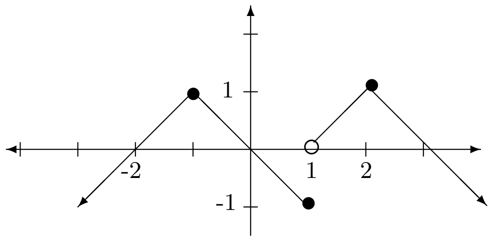
$$f'(x) = \frac{2(3+x)}{3(2+x)^{2/3}}, \qquad f''(x) = \frac{4(3+x)}{9(2+x)^{5/3}}.$$

a) For what values of x is $f(x)$ defined?

b) Determine the open interval(s) on which $f(x)$ is increasing.

c) Determine the open interval(s) on which the graph of $f(x)$ is concave up.

(12) 5. The graph of $y = f(x)$ is given below.



a) Find $\lim_{x \to 1^-} f(x) =$

b) Indicate all values of x at which $f(x)$ is not continuous.

c) Indicate all values of x at which $f'(x)$ is not defined.

d) Evaluate $f'(3) =$

e) Evaluate $\int_{-2}^{1} f(x)\, dx =$

(12) 6. A farmer plans to fence a rectangular pasture adjacent to a river. The pasture must contain180,000 square meters in order to provide enough grass for the herd. What dimensions would require the least amount of fencing if no fencing is needed along the river?

(10) 7. Evaluate the definite integral $\displaystyle\int_0^1 \frac{(t+2)\ dt}{\sqrt{t^2+4t+1}}$

(10) 8. Find the function $f(t)$, given that $f''(t) = 2,\ f'(2) = 1,$ and $f(2) = 7.$

(12) 9. A car A is travelling north (up) along the y-axis while a car B is travelling west (to the left) along the x-axis. At a particular instant, car A is at the point (0, 40) (mi) and travelling north at 50 mi/hr while car B is at (30,0) and travelling west at 30 mi/hr. At what rate is the distance between the two cars changing at this instant?

(14) 10. Evaluate the indefinite integrals.

a) $\int \sqrt{\cos x} \sin x \; dx =$

b) $\int (\sin(3t) + \sec(2t)\tan(2t)) \; dt =$

(11) 11. Find the linear approximation $L(x)$ to the function $f(x) = \sqrt{x}$ near $x = 9$.

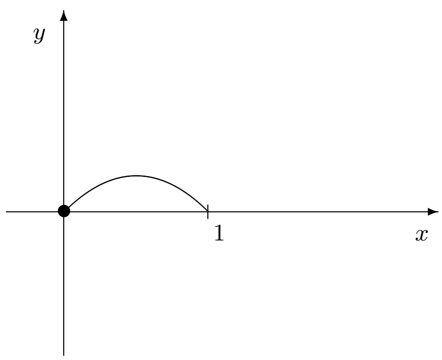
(10) 12. Differentiate using the second fundamental theorem of calculus:

a. $F'(x) = \dfrac{d}{dx} \displaystyle\int_1^x \frac{1}{t^2} dt =$

b. $G'(y) = \dfrac{d}{dy} \displaystyle\int_0^{\cos y} \sqrt{t} dt =$

(10) 13. Sketch the region bounded between the graphs of the functions $f(x) = x^2$ and $g(x) = x + 2$, and then calculate its area.

(14) 14. Below is a sketch of the region bounded by the curve $y = x - x^3$ and the x-axis over the interval $0 \leq x \leq 1$. Set up integrals for the following volumes but **do not evaluate the integrals:**



a) The volume of the solid obtained by revolving the region around the vertical line $x = -1$.

b) The volume of the solid obtained by revolving the region around the x-axis.

(14) 15. a) Set up the integral that gives the length of the arc $y = x^{4/3}$, $0 \leq x \leq 8$. **Do not evaluate the integral.**

b) Set up the integral that gives the surface area of the surface generated by revolving the arc in a) about the x-axis. **Do not evaluate the integral.**

(10) 16. Find the value(s) of c guaranteed by the Mean Value Theorem for Integrals for the function $f(x) = \dfrac{1}{x^2}$ over the interval $[1, 3]$.