Math 205 Final Exam

Wednesday December 17

Your name:____________________________

Instructor's name:________________________ **Time the class meets:**________________

Calculators are permitted. You can also use a 4×6 inches note card with formulas prepared in advance. No books are allowed. Use the back pages as sketch paper. For full credit, show your work in detail. If you use a calculator at some step, you should state that you are doing so.

Total: 200	Pg 1	Pg 2	Pg 3	Pg 4	Pg 5	Pg 6	Pg 7	Pg 8
200	20	33	26	28	25	23	23	22

1. (10 points) Let $f(x) = 2x^2$, and $g(x) = x + 2$. Find

(a) $f(3) + g(3) =$

(b) $f(3)g(3) =$

(c) $f(g(3)) =$

(d) $g(f(x)) =$

2. (10 points) The following figure shows cost and revenue for a company.

(a) What are the fixed costs?

(b) Approximately what quantity does this company have to produce to make a profit?

(c) Estimate the profit generated by 600 units.

3. (10 points) A product costs \$90 today. How much will the product cost in t days if the price is reduced by

(a) 4% a day.

(b) \$5 a day.

4. (10 points) Solve for t using natural logarithms:

$$3 \cdot 7^t = 2 \cdot 5^t.$$

5. (13 points) For a function given by the following table

t	80	90	100	110	120
$w(t)$	21.4	12.6	8.4	7.0	6.6

(a) does the derivative appear to be positive or negative over the given interval? Explain.

(b) does the second derivative appear to be positive or negative over the given interval? Explain.

6. (14 points) Find the derivative for the following functions

(a) x^2e^{-x}

(b) $(e^x + \ln(x))^{0.5}$

7. (12 points) For the following cost function, is marginal cost greater at $q = 5$ or at $q = 30$? At $q = 20$ or at $q = 40$? Explain.

8. (14 points) For each cost function below, graph the corresponding average cost function:

9. (14 points) A manufacturer sells two products, one at a price \$3 a unit and the other at a price of \$12 a unit. A quantity q_1 of the first product and q_2 of the second product are sold by the manufacturer. Assuming that the total cost is \$4,000 and does not dependent on the quantities q_1, q_2 produced,

(a) express the manufacturer's profit, π, as a function of q_1 and q_2.

(b) Sketch contours of π for $\pi = 10,000$, $\pi = 20,000$, $\pi = 30,000$ and the break-even curve $\pi = 0$.

10. (13 points) For the function $f(x, y) = \ln(x + y)$ find both partial derivatives f_x, f_y.

11. (12 points) Let $f(x, y) = 3xy + y^2 + 3x - 5$.

(a) Find the critical point (a, b) of the function $f(x, y)$.

(b) Apply the test function $D = f_{xx}(a, b)f_{yy}(a, b) - f_{xy}^2(a, b)$ and the sign of f_{xx} to determine whether the critical point is a local minimum, local maximum, or neither.

12. (13 points) Use the method of Lagrange multipliers to find the maximum value of $f(x, y) = 4xy + y^2$ subject to the constraint $x + y = 50$.

13. (10 points) Without using a calculator solve the following system of equations

$$\begin{cases} x + 3y &= 7 \\ x + y &= 3 \end{cases}$$

14. (12 points) Solve the following system of equations by finding the reduced row echelon form of the augmented matrix:

$$\begin{cases} x+y+z &= 5 \\ x-y+z &= 8 \\ 2x+2z &= 13 \end{cases}$$

15. (11 points) For the matrices

$$A = \begin{bmatrix} 5 & 1 \\ 1 & 1 \\ 0 & -1 \end{bmatrix} \qquad B = \begin{bmatrix} -3 & 0 \\ 0 & 0 \\ 1 & 1 \end{bmatrix} \qquad C = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

find

(a) $A+B$

(b) $(A+B)C$

(c) $C(A+B)$

16. (10 points) Find the inverse of the following matrix using the reduced row echelon technique.

$$\begin{bmatrix} 0 & 0 & 1 \\ 0 & 2 & 0 \\ 3 & 0 & 0 \end{bmatrix}$$

17. (12 points) Data giving the number of hours a person studied compared to his or her performance on an exam are given in the table:

Hours Studied x	0	4	8	12	16
Exam Score y	8	20	40	62	75

(a) Find a least squares line of best fit to the above data.

(b) What prediction would this line make for a student who studied 13 hours?