Math 205 Exam 3

Thursday April 16

Your name:_____

Instructor's name:_____ **Time the class meets:**_____

Calculators are permitted. You can also use a 4×6 inches note card with formulas prepared in advance. No books are allowed. Use the back pages as sketch paper. For full credit, show your work in detail. If you use a calculator at some step, you should state that you are doing so.

Total: 100	# 1	# 2	# 3	# 4	# 5	# 6	# 7	# 8	# 9	# 10
100	10	10	10	10	10	10	10	10	10	10

1. (10 points) When production is 2000, marginal revenue is \$4.25 per unit and marginal cost is \$3 per unit. Do you expect maximum profit to occur at a production level above or below 2000? Explain your answer.

2. (10 points) When production is 1000, the average cost is \$5 per unit and the marginal cost is \$4 per unit. If the production level increases how it will affect the average cost? Explain your answer.

3. (10 points) If t is in years since 1990, one model for the population of the world, P, in billions, is

$$P = \frac{40}{1 + 11e^{-0.08t}}$$

a. What this model predict for the maximal sustainable population?

b. Find the year when the population will be growing the fastest according to this model.

4. (10 points) The figure below shows contours of the function giving the species density of breeding birds at each point in the US, Canada, and Mexico. Are the following statements true or false? Explain your answers.

a. Moving from south to North across Canada, the species density increases.

b. The species density around Miami is over 100.

5. (10 points) The demand curve for a product is given by $q = 100 - p$. Find the elasticity of demand at a price $p = \$60$. Should the owner raise or lower the price in order to maximize the revenue?

6. (10 points) In each case, give a possible contour diagram for the function $f(x, y)$ if

a. $f_x > 0$ and $f_y > 0$.

b. $f_x > 0$ and $f_y = 0$.

c. $f_x > 0$ and $f_y < 0$.

7. (10 points) Find the partial derivatives for the following functions

a. $f(x, y) = 2x - 3y$.

b. $f(u, w) = ue^w$.

8. (10 points) For the function $f(x, y) = y/x$ calculate both first order partial derivatives and all four second order partial derivatives and confirm that the mixed partials are equal.

9. (10 points) a. Find the critical point (a, b) of the function $f(x, y) = x^2 + 4xy + y^2 + 6x + 1$.

b. Apply the test function $D = f_{xx}(a, b)f_{yy}(a, b) - f_{xy}^2(a, b)$ and the sign of $f_{xx}(a, b)$ to determine whether the critical point is a local maximum, local minimum, or neither.

10. (10 points) The figure below shows contours of $f(x, y)$ and the constraint $g(x, y) = c$. Approximately what values of x and y maximize $f(x, y)$ subject to the constraint? What is the approximate value of f at this maximum?