MATH 205 EXAM 4

Friday July 27, 2007

Instructor's Name:____________________ **Name:**__________________________

Calculators may be used sparingly, but no books or formula sheets are allowed. Use the back pages as sketch paper. For full credit, show your work in detail.

Total: 100	#1	#2	#3	#4	#5	#6	#7	#8

1 (12 pts). Solve each system of equations using either the substitution or the elimination method. If the system has no solution, state so.

(a) (4 pts).

$$\begin{cases} x + 3y &= 0 \\ 2x &= 12 \end{cases}$$

(b) (4 pts).

$$\begin{cases} 2x - y &= 0 \\ 3x + 2y &= 7 \end{cases}$$

(c) (4 pts).

$$\begin{cases} 2x - 3y &= -1 \\ 10x + y &= 11 \end{cases}$$

2 (12 pts). The reduced row echelon form of the augmented matrix of a system of linear equations is given. Tell whether the system has only one solution, no solution, or infinitely many solutions. Write the solutions or, if there is no solution, state that the system is inconsistent.

(a) (4 pts).

$$\left[\begin{array}{cc|c} 1 & 0 & 4 \\ 0 & 1 & 5 \end{array}\right]$$

(b) (4 pts).

$$\left[\begin{array}{ccc|c} 1 & 2 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{array}\right]$$

(c) (4 pts).

$$\left[\begin{array}{ccc|c} 1 & 0 & 0 & 1 \\ 0 & 1 & 0 & 2 \\ 0 & 0 & 0 & 0 \end{array}\right].$$

3 (12 pts). Solve the following systems of equations using the matrix method (write the augmented matrix and find the row echelon form or reduced row echelon form). If the system has no solution, state that the system is inconsistent.

(a) (6 pts).

$$\begin{cases} x + y - z &= 0 \\ 3x - y &= 1 \\ 2x - 3y + 4z &= 8 \end{cases}$$

(b) (6 pts).

$$\begin{cases} x - y + z &= 5 \\ x - 2y + 2z &= 2 \\ 2x + y - z &= 1 \end{cases}$$

4 (12 pts). You do not need to use a calculator to solve this problem. Please show all work.

(a) (6 pts). Verify that

$$\begin{bmatrix} 1 & 1 & 2 \\ 2 & 1 & 0 \\ 1 & 2 & 2 \end{bmatrix}^{-1} = \begin{bmatrix} \frac{1}{2} & \frac{1}{2} & -\frac{1}{2} \\ -1 & 0 & 1 \\ \frac{3}{4} & -\frac{1}{4} & -\frac{1}{4} \end{bmatrix}$$

(*HINT:* If A and B are square matrices having the same dimensions, then $A^{-1} = B$ if and only if the matrix product AB is an identity matrix.)

(b) (6 pts). Using part (a), solve the linear system:

$$\begin{cases} x + y + 2z &= 4 \\ 2x + y &= 12 \\ x + 2y + 2z &= 16 \end{cases}$$

5 (12 pts). Find the inverse of

$$\begin{bmatrix} 1 & 2 \\ 2 & 3 \end{bmatrix}$$

using the reduced row echelon technique.

6 (12 pts). Find the partial derivatives f_x and f_y for each function below.

(a) (4 pts). $f(x, y) = x^2 + 2xy + y^3$

(b) (4 pts). $f(x, y) = 100x^2y$

(c) (4 pts). $f(x, y) = 10x^2e^{3y}$

7 (16 pts). For the function below, find all critical points and classify each as a local maximum, local minimum, or saddle point.

$$f(x, y) = 2x^3 - y^3 - 3x^2 + 3y + 12$$

8 (12 pts). Use the method of Lagrange multipliers to find the maximum or minimum values of $f(x, y)$ subject to the constraint $g(x, y) = 2$, where $f(x, y) = x + y$ and $g(x, y) = x^2 + y^2$.