Math 205 Exam 3

Friday July 14, 2006

Instructor:_____ **Class Time:**_____ **Name:**_____

No books or notes are allowed. Please read the problems carefully and do **all** you are asked to do. **You must show your work!** You can use the back page as a scratch paper. Do the problems at the space provided.

Total/100	#1	#2	#3	#4	#5	#6	#7	#8

1(total points: 12). Given the function $f(x) = xe^{-x}$:

(a)**(6 points).** Find the critical points of f.

(b) **(6 points).** Find the inflection points of f.

2 (total points: 12 points). Find the global maxima and minima of $f(x) = xe^{-.5x}$ on

(a) **(8 points).** $-5 \leq x \leq 10$

(b) **(4 points).** $5 \leq x \leq 20$

3 (total points: 12). The **average** cost in thousands of dollars per unit of a good is

$$a(q) = 0.2q^2 - 12q + 260,$$

where q is the number of units produced.

(a) **(5 points).** Find the formula for the marginal cost when q units are produced.

(b) **(7 points).** Find the minimum average cost.

4 (total points: 12). A total cost function of a company for producing q units of some product is given by $C(q) = 0.00001q^3 - 0.06q^2 + 150q + 100000$, in dollars, where $0 \leq q \leq 5000$.

(a)**(4 points).** If the company can sell the product at the price of 150 dollars per unit, find a formula for the profit function.

(b) **(6 points).** Find the production level that maximizes the profit.

(c) **(3 points).** What is the maximum profit?

5 (total points: 14). A publisher printed 190,000 copies of a new book. Suppose that t days since the release of the book,

$$Q(t) = \frac{90000}{1 + 299e^{-0.1t}}$$

copies were bought. That is, $Q(t)$ is a logistic function.

(a) **(4 points).** How many copies were bought immediately after its release?

(b) **(6 points).** When was the book sold at the peak speed?

(c) **(4 points).** How many copies, if any, of the book will never be bought?

6 (total points: 13). Suppose that the demand for coffee, Q, in thousands of pounds per week, depends on the price of coffee, c, and the price of tea, t, both in dollars per pound, according to the formula

$$Q = f(c, t) = 100\frac{t}{c}. \tag{1}$$

(a) **(7 points).** Make a table showing the value of Q when $t = 1, 2, 3, 4$ and $c = 1, 2, 3, 4$.

(b) **(3 points).** Is Q an increasing or decreasing function of c?

(c) **(3 points).** Is Q an increasing or decreasing function of t?

7 (total points: 13). An ice cream company finds that at a price of \$4.00, demand is 4000 units. For every \$0.25 decrease in price, demand increases by 200 units.

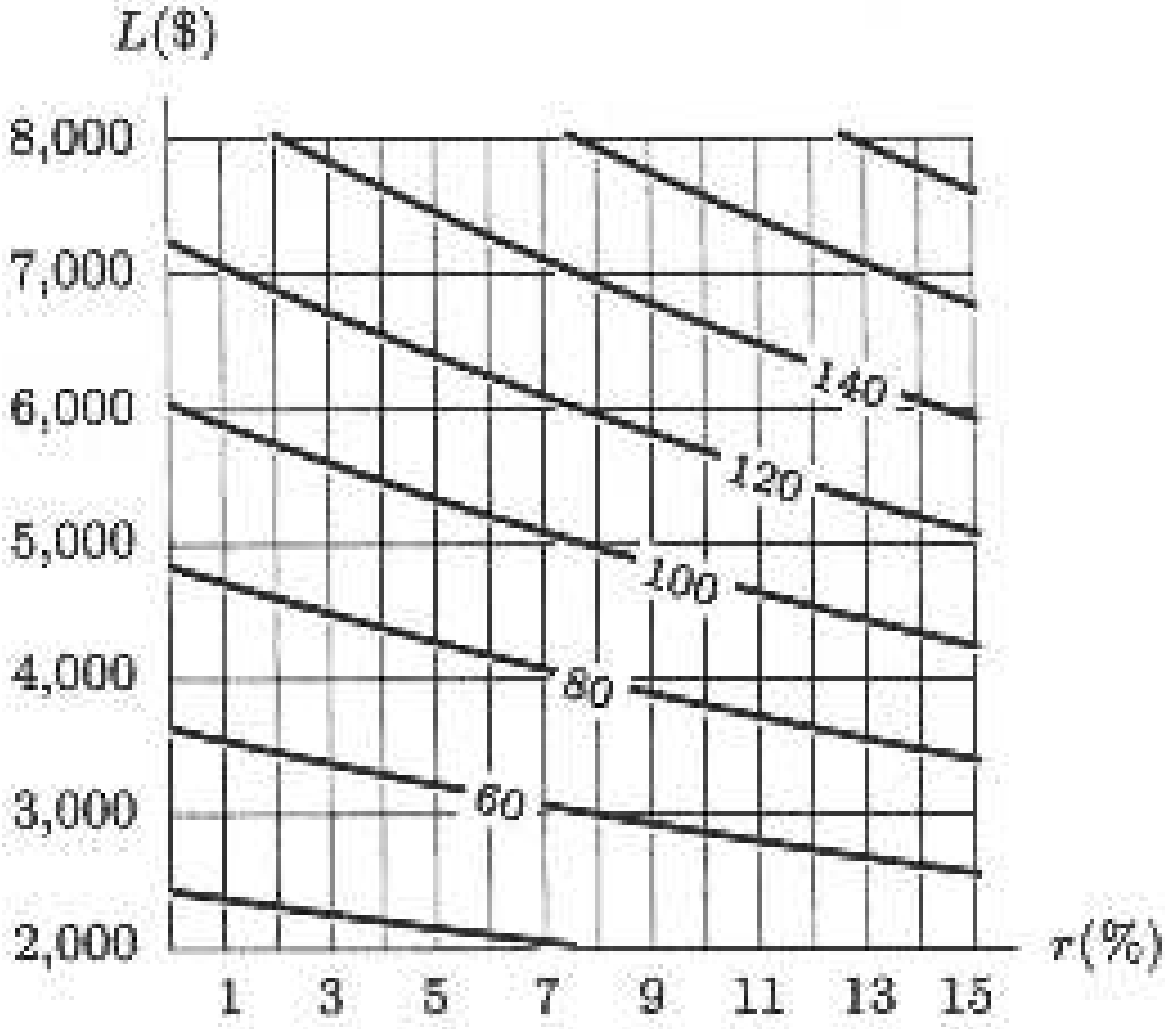
(a) **(6 points).** Express the demand q in terms of the price p. (Hint: q is a linear function of p.)

(b) **(7 points).** Find the elasticity of demand at the current price of \$4.00. Should the company increase the price in order to increase the revenue?

8 (total points: 12). The figure below shows a contour diagram for the monthly payment P as a function of the interest rate, $r\%$ and the amount L, of a 5-year loan. The interest rate is 13% and you borrow \$6,000.

(a) **(6 points).** What is your monthly payment.

(b) **(6 points).** If interest rates drop to 11%, how much more can you borrow without increasing your monthly payment?



(c) **(6 points).** Make a table of how much you can borrow, without increasing your monthly payment, as a function of the interest rate.