Math 205 Final Exam

Friday July 28, 2006

Instructor:_____ **Class Time:**_____ **Name:**_____

No books or notes are allowed. Please read the problems carefully and do **all** you are asked to do. **You must show your work!** You can use the back page as a scratch paper. Do the problems at the space provided.

Total/100	#1	#2	#3	#4	#5	#6	#7	#8	#9

1 (total points: 15). The table below gives the number of calories burned per minute, $B = f(w, s)$, for someone roller-blading, as a function of the person's weight w, and the speed, s.

$w \backslash s$	8 mph	9 mph	10 mph	11 mph
120 lbs	4.2	5.8	7.4	8.9
140 lbs	5.1	6.7	8.3	9.9
160 lbs	6.1	7.7	9.2	10.8
180 lbs	7.0	8.6	10.2	11.7
200 lbs	7.9	9.5	11.1	12.6

Table 1: Calories burned per minute

(a) **(10 points).** Estimate $f_w(160, 10)$.

(b) **(5 points).** Interpret your answer.

2 (total points: 30). Given $f(r, t) = \ln(r - t) + 100e^{0.2rt}$, find the partial derivatives below: (Simplify your answers.)

(a)**(10 points).**

$f_r(r, t) =$

(b)**(10 points).**

$f_t(r,t) =$

(c)**(10 points).**

$f_{rt}(r,t) =$

3 (total points: 20 points). Let $f(x,y) = x^2 + y^2 + 6x - 10y + 8$.

(a) **(10 points).** Find all the critical points of the function $f(x,y)$.

(b) **(10 points).** Use the second derivative test involving the test function $D(x,y) = f_{xx}f_{yy} - (f_{xy})^2$ to determine whether each critical point is a local maximum, local minimum, or neither.

4 (total points: 24).

(a) **(16 points).** Use the method of Lagrange multipliers to find the minimum or maximum values of $f(x, y) = x^2 + 3y^2 + 100$ subject to the constraint $8x + 6y = 88$. Show which ones are maxima and which ones are minima.

(b) **(8 points).** Determine the value of the parameter λ and interpret it.

5 (total points: 21). Find x, y and z so that

$$\begin{bmatrix} x-2 & 3 & 2z \\ 6y & x & 2y \end{bmatrix} = \begin{bmatrix} y & z & 6 \\ 18z & y+2 & 6z \end{bmatrix}$$

6 (total points: 21). Let $A = \begin{bmatrix} -2 \\ 0 \end{bmatrix}$, and $B = \begin{bmatrix} 1 & -20 \\ 5 & 100 \end{bmatrix}$. Perform each of the following operations or indicate that the operation is not defined.

(a) **(7 points).** $A + B =$

(b) **(7 points).** $BA =$

(c) **(7 points).** $AB =$

7 (total points: 24). For each of the following augmented matrices, check the appropriate box to indicate if the corresponding linear system has a unique solution, has no solution or has infinitely many solutions.

(a) **(8 points).** $\left[\begin{array}{ccc|c} 1 & 2 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{array}\right]$

has a unique solution ..... ☐,
has no solution ..... ☐,
has infinitely many solutions ..... ☐.

(b) **(8 points).** $\left[\begin{array}{ccc|c} 1 & 0 & 0 & -1 \\ 0 & 1 & 0 & 3 \\ 0 & 0 & 1 & 4 \\ 0 & 0 & 0 & 0 \end{array}\right]$

has a unique solution ..... ☐,
has no solution ..... ☐,
has infinitely many solutions ..... ☐.

(c) **(8 points).** $\left[\begin{array}{cccc|c} 1 & 0 & 2 & 4 & -1 \\ 0 & 1 & 3 & 5 & -2 \end{array}\right]$

has a unique solution ..... ☐,
has no solution ..... ☐,
has infinitely many solutions ..... ☐.

8 (total points: 20). Solve the following system of equations by the inverse matrix method (*no credit for other methods!*):

$$\begin{cases} x + y + 2z &= 1 \\ 2x + y &= 2 \\ x + 2y + 2z &= 3 \end{cases}.$$

Note that $\begin{bmatrix} 1 & 1 & 2 \\ 2 & 1 & 0 \\ 1 & 2 & 2 \end{bmatrix}^{-1} = \begin{bmatrix} \frac{1}{2} & \frac{1}{2} & -\frac{1}{2} \\ -1 & 0 & 1 \\ \frac{3}{4} & -\frac{1}{4} & -\frac{1}{4} \end{bmatrix}$.

9 (total points: 25). Write out the augmented matrix of the following system of equations, and then find the reduced row-echelon form of the augmented matrix to solve the following system of linear equations

$$\begin{cases} x + y - z &= 5 \\ 2x - y - 2z &= 2 \\ x + y + z &= 3 \end{cases}.$$

You must provide details of your work step by step.