MATH 205 FINAL EXAM

7:00–8:50 pm, Wednesday May 9, 2007

Instructor's Name:____________________ **Name:**__________________________

No books or formula sheets are allowed. Use the back of the page as sketch paper. For full credit, show your work in detail.

Total 200	Pg 1	Pg2	Pg3	Pg4	Pg5	Pg6	Pg7	Pg8

1 (13 pts). A company has cost and revenue function, in dollars, given by $C(q) = 6000 + 10q$ and $R(q) = 12q$.

- **a (6pts).** Write out the profit function $P(q)$, and compute $P(5000)$.

- **b (7pts).** Find the break-even point q. (Do not draw graph.)

2 (12 pts). Sketch the derivative function of the given function in the same cooridinate system.

3 (15 pts). Find derivatives of following functions.

a. $y = 8t^3 - 4t^2 + 12t - 3$

b. $s = 5e^{\sqrt{x}}$

c. $w = \dfrac{3y + y^2}{5 + y}$.

4 (10 pts). If t is in hour, the concentration of the caffeine, in mg, from a cup of coffee remains in blood is given by $C(t) = 5te^{-0.2t}$. In how many hours does the concentration of caffeine reach its maximum?

5 (13 pts). The demand of quantity for mango (a tropical fruit) is given by $q = 1500 - 20p^2$ where q is in pound and p, in dollar, is the price of mango/per pound.

(a). Find the revenue function R in terms of the price p.

(b). What is the price p would you recommend to maximize the revenue?

6 (12 pts). For the function $f(x) = 4 \ln x - x$, and $3 \leq x \leq 5$, find the global minimal and global maximal values of $f(x)$, and indicate where they are reached.

7 (15 pts). Find partial derivatives of following functions.

(a). Let $f(x,y) = x^3 + 3x^2y - 2y^2$. Find $f_x(1,2)$ and $f_y(1,2)$.

(b). $f(x,y) = \dfrac{2x}{y}$. Find f_{xx}, f_{xy}, f_{yy}.

8 (10 pts). For a function $f(x,y)$, it is known that $f(100,20) = 2750$, $f_x(100,20) = 4$ and $f_y(100,20) = 7$. Estimate $f(103,25)$.

9 (10 pts). Apply the method of Lagrange multipliers to solve following problem. A company manufactures x units of one item and y units of another. The total cost (in dollars) is given by $C(x, y) = 2x^2 + 2xy + 3y^2 + 500$.

If the total number of items (both types combined) is 45, find the quota x and y minimizing the cost.

10. (15pts.) For each of the following augmented matrices, indicate whether the corresponding system has a unique solution, no solution, or infinitely many solutions.

(a.) $\left[\begin{array}{ccc|c} -2 & 1 & 5 & 1 \\ 2 & 8 & 7 & 3 \\ 0 & 0 & 0 & -1 \end{array}\right]$

(b.) $\left[\begin{array}{ccc|c} 4 & 7 & -5 & 9 \\ 0 & 5 & 2 & -1 \\ 0 & 0 & 0 & 0 \end{array}\right]$

(c.) $\left[\begin{array}{ccc|c} 3 & 8 & 5 & -2 \\ 0 & 1 & 3 & 7 \\ 0 & 0 & 1 & 6 \end{array}\right]$

10.(13pts). Find x, y and z so that

$$\begin{bmatrix} 3x+1 & 3 \\ 5 & x+2 \\ 4 & z+5 \end{bmatrix} = \begin{bmatrix} y+2 & y+1 \\ y+z & z \\ 4 & 4y \end{bmatrix}.$$

12 (13pts). Let $A = \begin{bmatrix} 2 & -3 & 4 \\ 0 & 2 & 1 \end{bmatrix}$, $B = \begin{bmatrix} 1 & 2 & 0 \\ 5 & 1 & 2 \end{bmatrix}$, $C = \begin{bmatrix} -3 & 0 & 5 \\ 2 & 1 & 3 \end{bmatrix}$.

Find $2A - 5(B + C)$

13 (13 pts). Solve the following system of linear equations:

$$\begin{cases} x & +2y & -2z & = & 1 \\ 2x & +5y & +z & = & 9 \\ x & +3y & +4z & = & 9 \end{cases}$$

(1). Write out the augmented matrix AUG.

(2). Find the row reduction echelon form rref either manually or by a calculator.

(3). Find solutions x, y, z.

14. 12 pts. Find the product $\begin{bmatrix} 4 & 0 & 6 \\ 1 & -2 & 3 \end{bmatrix} \begin{bmatrix} -1 & 2 & 1 \\ 1 & 3 & 0 \\ 0 & 4 & -2 \end{bmatrix}$

15 (13pts). It is given that the inverse $\begin{bmatrix} -1 & 1 & 1 \\ 16 & -14 & -13 \\ 6 & -5 & -5 \end{bmatrix}^{-1} = \begin{bmatrix} 5 & 0 & 1 \\ 2 & -1 & 3 \\ 4 & 1 & -2 \end{bmatrix}$. Make use of the above information to solve following linear system:

$$\begin{cases} -x & + & y & + & z & = & 6 \\ 16x & - & 14y & - & 13z & = & 2 \\ 6x & - & 5y & - & 5z & = & 7. \end{cases}$$

16 (12pts). A closed economy system consists of three sectors A, B, C. The input–output matrix is given by

\	A	B	C
A	0.4	0.3	0.2
B	0.2	0.3	0.3
C	0.4	0.4	0.5

.

Find the salaries $X = [x, y, z]$ for sectors A, B and C. (You may set the $z = \$30,000$.)