Math 205 Final Exam

7:00p.m.–8:50p.m., Wednesday, May 10, 2006

Instructor:_____ **Class Time:**_____ **Name:**_____

No books or notes are allowed. Please read the problems carefully and do **all** you are asked to do. **You must show your work!** You can use the back page as a scratch paper. Do the problems at the space provided.

Total/200	#1	#2	#3	#4	#5	#6	#7	#8	#9	#10	#11	#12	#13	#14

1 (total points: 12). A $\$20,000$ car depreciates *linearly* down to $\$4,000$ in 8 years.

(a)**(6 points)** Find the formula for the value y of the car after x years for $0 \leq x \leq 8$.

(b)**(6 points)** After how many years is the value of the car the half of its original price?

2 (total points: 15). Find the derivative of the following functions:

(a)**(5 points)** $y = \ln(1 + 3x^2)$.

(b)**(5 points)** $t = s^3 \cdot e^{s-1}$.

(c)**(5 points)** $w = \dfrac{5u^2}{1 - 2u}$.

3 (total points: 15). The demand and supply curves for a certain product are given in terms of price, p (in dollars), by

$$D(p) = 2500 - 20p \qquad \text{and} \qquad S(p) = 10p - 500.$$

(a)**(6 points)** Find the equilibrium price and quantity.

(b)**(6 points)** If a specific tax of \$6 per unit is imposed on suppliers, find the new equilibrium price and quantity.

(c)**(3 points)** How much of the \$6 tax is paid by the suppliers and by the consumers, respectively?

4 (total points: 14). A person likes to put $\$3,000$ on a saving account. Bank A is paying a 3% nominal annual rate compounded every three months. Bank B is offering a 2.9% nominal annual rate compounded continuously.

(a)**(7 points)** Determine the amount in the account after 5 years for bank A.

(b)**(7 points)** How long does it take the balance in the account of bank B to double?

5 (total points: 12). The following information about a function $f(x, y)$ is given: $f(-10, 3) = 7.5$, $f_x(-10, 3) = 0.3$, and $f_y(-10, 3) = -1.2$. Estimate $f(-10.5, 3.4)$.

6 (total points: 15).

(a)**(5 points)** Find the partial derivative $\dfrac{\partial f}{\partial y}$ of $f(x, y) = 3x^4y^2 - x^2y + 7$.

(b)**(5 points)** Find the partial derivative g_x of $g(x, y) = \dfrac{\sqrt{x}}{y^2}$.

(c)**(5 points)** Find the partial derivative h_{st} of $h(s, t) = \ln(s + t)$.

7 (total points: 16). The total cost in dollars, $C(q)$, of producing q commercial dish washers is given by $C(q) = q^3 - 66\,q^2 + 1,500\,q + 10,000$.
(a)**(3 points)** What is the fixed cost?

(b)**(3 points)** Determine the profit function, $\pi(q)$, if each washer sells for \$1500.

(c)**(10 points)** What is the maximum total profit?

8 (total points: 14). (a)**(7 points)** Find all the critical points (a, b) of the function

$$f(x, y) = 3x^2 + y^2 + 4xy - 6x - 4371.$$

(b)**(7 points)** Use the D-test to decide for each of the critical points (a, b) found in part (a) if it is a local minimum, a local maximum, or neither. ($D = f_{xx}(a, b)f_{yy}(a, b) - f_{xy}^2(a, b)$.)

9 (total points: 15). A company manufactures x units of item A_1 and y units of item A_2. The total cost in dollars, C, of producing these two items is approximated by the function $C = 5x^2 + 2xy + 3y^2 + 800$.

(a)**(10 points)** If the production quota for the total number of items (both types combined) is 39, find the minimum production cost by using the Lagrange multiplier method.

(b)**(5 points)** Estimate the additional production cost if the production is raised to 41.

10 (total points: 15). Find the inverse of the matrix

$$A = \begin{bmatrix} 1 & 1 & 0 \\ 1 & 0 & 1 \\ 3 & 1 & 1 \end{bmatrix}$$

using basic row-operations.

11 (total points: 12). Find all values for x, y and z such that the following matrix equation is satisfied:

$$\begin{bmatrix} 10 & x+y+8 \\ -z & x-y \end{bmatrix} = \begin{bmatrix} 10 & 2x+3y \\ z^2 & y \end{bmatrix}.$$

12 (total points: 15). Given are the matrices $A = \begin{bmatrix} 1 & 2 & -3 \\ 4 & -5 & 6 \end{bmatrix}$, $B = \begin{bmatrix} 4 & 0 & -1 \\ 7 & 1 & 0 \end{bmatrix}$ and $C = \begin{bmatrix} -3 & 3 \\ 1 & 3 \\ 1 & 0 \end{bmatrix}$.

Compute the following matrix expressions or indicate that this is impossible:

(a)**(5 points)** $8A - 4(B + A) =$

(b)**(5 points)** $A(-C) =$

(c)**(5 points)** $C(-B) =$

13 (total points: 12). Solve the following system of equations by the inverse matrix method (*no credit* for other solutions!):

$$\begin{cases} x + y - z = 18 \\ 3x - y = -27 \\ 2x - 3y + 4z = 9 \end{cases}.$$

The inverse matrix of $A = \begin{bmatrix} 1 & 1 & -1 \\ 3 & -1 & 0 \\ 2 & -3 & 4 \end{bmatrix}$ is $A^{-1} = \begin{bmatrix} 4/9 & 1/9 & 1/9 \\ 4/3 & -2/3 & 1/3 \\ 7/9 & -5/9 & 4/9 \end{bmatrix}$.

14 (total points: 18). The following table describes the interrelationship between the production of two industries P and Q as well as the external demand of their products in a given year.

	Demand of P	Demand of Q	External Demand	Total Output
Production of P	6	30	24	60
Production of Q	30	40	50	120

If the forecast for the external consumer demand in the next year is $D' = \begin{bmatrix} 57 \\ 57 \end{bmatrix}$ what should the *future total output* $X' = \begin{bmatrix} x' \\ y' \end{bmatrix}$ be?