Math 205 Final Examination

7:00–8:50 PM, Wednesday, May 11, 2005

Instructor:_____ **Class Time:**_____ **Name:**_____

No books or notes are allowed. Please read the problems carefully and do **all** you are asked to do. **You must show your work!** You can use the back page as a scratch paper. Do the problems at the space provided.

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1(12 total points). Suppose that the interest of a savings account is compounded quarterly. If the nominal annual interest rate is 5%, find the effective annual yield.

2(13 total points). A function $f(x, y)$ satisfies $f(3, 5) = 40$, $f_x(3, 5) = 5$, and $f_y(3, 5) = 8$. Estimate $f(2.8, 5.5)$.

3(12 total points). Find the derivatives of the following functions. (Do not simplify)

(a)(4 pts). $y = \dfrac{e^x}{2+x}$.

(b)(4 pts). $f(x) = \ln(1 + e^x)$.

(c)(4 pts). $f(t) = t^2 \ln t$

4(13 total points). Find the global maximum and global minimum of the function $f(x) = x^3 - 3x$ over the interval $0.5 \leq x \leq 3$ (Must show your work to get any credit).

5(12 total points). Find the quantity which maximizes the profit if the total revenue and total cost (in dollars) are given by

$$R(q) = 5q - 0.003q^2$$

$$C(q) = 300 + 1.1q$$

where q is quantity and $0 \leq q \leq 1000$ units.

6(13 total points). Find the indicated partial derivatives of the following functions:

(a)(8pts). Given $f(x, y) = 10x^2e^{3y}$, find

$f_x(x, y) =$

$f_y(x, y) =$

(b)(5 pts). Given $f(x, t) = t^3 - 4x^2t$, find

$f_{xt} =$

7(13 total points). The total cost of production, in thousands of dollars, is $C(q) = q^3 - 12q^2 + 60q$, where q is in thousands and $0 \leq q \leq 8$.

(a)(5 pts). Find the average cost function $A(q)$.

(b)(8 pts) Find the value of q at which the average cost is minimized. What is the minimum average cost?

8(12 total points). Let $f(x, y) = x^2 - 2x + y^2 - 4y + 5$.

(a)(6 pts). Find the critical point of f;

(b)(6 pts). Use the second derivative test to determine if f has a local maximum, a local minimum, or neither at the critical point.

9(13 total points). Use Lagrange multipliers to find the minimum value of $f(x, y) = xy$ subject to the constraint $x + 2y = 12$.

10(12 total points). Suppose that $2A + B = C$ where

$$A = \begin{bmatrix} x & y \\ y & 8 \end{bmatrix}, \quad B = \begin{bmatrix} y & 3 \\ x & -6 \end{bmatrix}, \quad \text{and} \quad C = \begin{bmatrix} 5 & y+6 \\ 4 & -4 \end{bmatrix}.$$

Find x and y.

11(12 total points). Let $A = \begin{bmatrix} -2 & -1 \\ 1 & 0 \end{bmatrix}$, and $B = \begin{bmatrix} 1 & -1 \\ -1 & 1 \\ 0 & 1 \end{bmatrix}$. Perform each of the following operations or indicate that the operation is not defined.

(a)(6pts). $AB =$

(b)(6pts). $BA =$

12(12 total points). For each of the following augmented matrix, check the appropriate box to indicate if the corresponding linear system has a unique solution, has no solution, or has infinitely many solutions.

(a). (5pts) $\left[\begin{array}{ccc|c} 1 & 9 & 2 & 9 \\ 0 & 1 & 3 & -1 \\ 0 & 0 & 0 & 0 \end{array}\right]$

has a unique solution ..... ☐,
has no solution ..... ☐,
has infinitely many solutions ..... ☐.

(b). (5pts) $\left[\begin{array}{ccc|c} 1 & 0 & -3 & 3 \\ 0 & 1 & 2 & 0 \\ 0 & 0 & 5 & 0 \end{array}\right]$

has a unique solution ..... ☐,
has no solution ..... ☐,
has infinitely many solutions ..... ☐.

(c). (5pts) $\left[\begin{array}{ccc|c} 1 & -1 & 0 & 1 \\ 0 & 0 & 0 & 2 \\ 0 & 2 & 0 & -2 \\ 0 & 0 & 1 & 0 \end{array}\right]$

has a unique solution ..... ☐,
has no solution ..... ☐,
has infinitely many solutions ..... ☐.

13(13 total points). Use the reduced row-echelon technique to find the inverse of the matrix $A = \begin{bmatrix} 1 & -1 \\ 3 & -4 \end{bmatrix}$.

14(13 total points). Use the fact that the matrix

$A = \begin{bmatrix} -\frac{5}{2} & 2 & -\frac{1}{2} \\ 1 & -1 & 1 \\ \frac{1}{2} & 0 & -\frac{1}{2} \end{bmatrix}$ has the inverse $A^{-1} = \begin{bmatrix} 1 & 2 & 3 \\ 2 & 3 & 4 \\ 1 & 2 & 1 \end{bmatrix}$

to solve the linear system

$$\begin{cases} -\frac{5}{2}x + 2y - \frac{1}{2}z = 1 \\ x - y + z = 0 \\ \frac{1}{2}x - \frac{1}{2}z = -1 \end{cases}$$

No credit will be granted if the given A^{-1} is not used.

15(12 total points). The planning team of a company estimates that the total revenue, in thousands of dollars, of selling a certain product in a region for t months can be modeled by the logistic function

$$P = \frac{120}{1 + 15e^{-0.12t}}.$$

(a)(5 pts). What is the eventual revenue if the company keeps selling the product forever?

(b)(7 pts). When is the revenue growing most rapidly?

16(13 total points). Assume that 3 persons A, B, and C work for each other and themselves. Their work and wages are modeled by closed Leontief model with the input-output matrix

	A	B	C
A	1/2	1/4	2/3
B	1/4	1/2	1/6
C	1/4	1/4	1/6

Assume also that $z = C$'s wages $= \$45,000$. Find each person's wages.