Name(Print) _____ Lecture Time_____

Signature _____ Instructor_____

General Calculus - MATH 205, Fall 2007
Final Exam, Fall 2007

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points																	

1. **Show the details of your work under each question.**
2. **Clearly label your answer.**
3. **Each problem is 10 points except for #4, 8, 12, 16, which are 20 points each.**

1. Find the equation of the tangent line of $y = \ln x$ at $x = 3$.

2. A company has cost and revenue functions, in dollars, given by $C(q) = 1000 + 8q$, and $R(q) = 12q$.

(a) Find out whether the company makes a profit if it produces 500 units.

(b) Find the break-even point

(c) Illustrate the break-even point in graph.

3. The population of a town is 1000 at time $t = 0$.

(a) Write a formula for the population, P, as a function of year t if the population increases by 55 people a year.

(b) Write a formula for the population, P, as a function of year t if the population increases by 4.5% a year. (Hint: it is an exponential function of the form $P = P_0 a^t$.)

(c) Rewrite the formula in part (b) in the form of $P = P_0 e^{kt}$.

(d) Using the formula from part (b), estimate the population after 10 years.

4. A health club has cost and revenue functions given by $C = 10000 + 35q$ and $R = pq$, where q is the number of annual club members and p is the price of a one-year membership. The demand function for the club is $q = 3000 - 20p$.

(a) Write the cost and revenue as functions of p.

(b) Graph the cost and revenue as functions of p on the same axes.

(c) For what prices does the club make a profit?

(d) Estimate the annual membership fee that maximizes the profit.

5. (a) Suppose f has a continuous **derivative** whose values are given in the following table.

x	0	1	2	3	4	5	6	7	8	9	10
$f'(x)$	5	2	1	-2	-5	-3	-1	2	3	1	-1

Estimate the x-coordinates of critical points of f for $0 \leq x \leq 10$, and for each of the critical point, indicate if it is a local maximum of f, or local minimum, or neither.

(b) Suppose f has a continuous **derivative** whose values are given in the following table.

x	0	1	2	3	4	5	6	7	8	9	10
$f(x)$	5	2	1	-2	-5	-3	-1	2	3	1	-1

Estimate the x-coordinates of critical points of f for $0 \leq x \leq 10$, and for each of the critical point, indicate if it is a local maximum of f, or local minimum, or neither.

6. Find the global max and min of $f(x) = x - \ln x$, $1/2 \leq x \leq 2$.

7. (a) $f(x) = x^{4/3} + \sqrt{x},\ f'(x) =$

(b) $f(x) = \frac{1}{\sqrt[4]{x}},\ f'(x) =$

c $f(x) = e^x(\ln x + x) + x^4,\ f'(x) =$

(d) $f(x) = e^{x^2+3},\ f'(x) =$

8. The strength of a drug in the bloodstream is given function $f(t) = te^{-t}$. The time range is from $t = 0$ to $t = 10$ hours. Find the times when the drug has maximum strength and minimum strength.

9. The following table gives the number of calories burned per minute, $B = f(s, w)$, for someone roller-blading, as a function of the person's weight w, and the speed s.

(a) Is f_w positive or negative? How about f_s?

(b) Estimate $f_w(160, 10)$ and $f_s(160, 10)$.

10. Find the values of x and y which maximize $f(xy) = 400 - 3x^2 - 4x + 2xy - 5y^2 + 48y$. Explain the details.

(Hint: the second derivative test claims that, let $D = f_{xx}(x_0, y_0) f_{yy}(x_0, y_0) - f_{xy}(x_0, y_0)^2$. (1) if $f_{xx}(x_0, y_0) > 0$, $D > 0$, then f has a local min at (x_0, y_0); (2) if $f_{xx}(x_0, y_0) < 0$, $D > 0$, then f has a local max at (x_0, y_0).)

11 Find the following partial derivatives for $f(x, y) = 10x^2e^y + x - y$.

(a) $f_x =$

(b) $f_y =$

(c) $f_{xy} =$

(d) $f_{yy} =$

12 Use Lagrange multipliers to find the maximum or minimum values of $f(x, y) = x^2 + 3y^2 + 100$ subject to constraint $8x + 6y = 88$.

(Hint: equations you may need to use: $f_x = \lambda g_x$, $f_y = \lambda g_y$, $g = c$.)

13 Solve the system of equations by both methods: elimination and substitution.

$$\begin{cases} x - 2y &= 3 \\ 3x + 4y &= 5 \end{cases}$$

(a) By elimination.

(b) By substitution.

14 Given a system of equations

$$\begin{cases} x + 2y &= a \\ 3x + by &= 2 \end{cases}$$

(a) Give an example of values of a, b such that the above system of equations has no solution. Explain why.

(b) Give an example of values of a, b such that the above system of equations has infinite many solutions. Explain why.

15 Given $A = \begin{pmatrix} 1 & 2 & -3 \\ 4 & -5 & 6 \\ -7 & 8 & 9 \end{pmatrix}$, $B = \begin{pmatrix} 3 & 2 & 1 \\ -6 & -5 & -4 \\ 7 & -8 & -9 \end{pmatrix}$, and $C = \begin{pmatrix} 0 & 0 & 1 \\ 1 & 0 & 0 \\ 0 & 0 & 1 \end{pmatrix}$

(a) Find $2A + 3C =$

(b) Find $(2A + B)C =$

16 (a) First rewrite the system of equations

$$\begin{cases} x + 2y + 3z &= 0 \\ 3x + 4y - 5z &= 1 \\ 5x + 8y + 2z &= 2 \end{cases}$$

into the following form $AX = B$, where A is a 3×3 matrix, X and B are 3×1 matrices.

(b) Find the row echelon form of augmented matrix $[A|B]$, which is a 3×4 matrix now. Then use this form to solve the system of equations to get $x =?, y =?, z =?$.

(c) Find the inverse A^{-1} of the matrix A. Then use the formula $X = A^{-1}B$ to solve the system of equations to get $x =?, y =?, z =?$.