MATH 205 FINAL EXAM

7:00–8:50 pm, Wednesday December 13, 2006

Instructor's Name:_____ **Name:**_____

No books or formula sheets are allowed. Use the back of the page as sketch paper. For full credit, show your work in detail.

Total 200	Pg 1	Pg2	Pg3	Pg4	Pg5	Pg6	Pg7	Pg8

1 (12 pts). The quantities of demand and supply functions for a certain product are given in terms of price, p, by $D(p) = 2500 - 20p$ and $S(p) = 10p - 500$. Find the the equilibrium price p and quantity.(You need not draw a graph.)

2 (13 pts). Find the derivative of the function $y = 4x^3 + 3x^2 - 5x$ at $x = 2$.

3 (15 pts). Find derivatives of following functions.

a. $y = z^2 + \dfrac{1}{z}$

b. $s = 5t + \ln(t + 2)$

c. $w = \dfrac{3z - 1}{5 + z}$.

4 (10 pts). Find the equation of the tangent line to the function of $f(x) = x^3 + 1$ at $x = 1$. (Do not draw the graph)

5 (15 pts). The total cost of the production of q units of certaing product is $C(q) = 10,000 + 3q^2$, and the product sells for 450 per unit.

- (1). Find the revenue function $R(q)$ and profit function $P(q)$.

- (2). At what quatity q is profit function $P(q)$ maximized? (Show your work in details.)

6 (10 pts). Find partial derivatives of following functions:

(a. 5pts). $\dfrac{\partial Q}{\partial x}$ if $Q(x, y) = 5x^2y - 3xy^3$

(b. 5pts). f_x if $f(x, y) = 5e^{xy}$.

7 (12 pts). The following table gives some data of a function $f(x, y)$:

$y \setminus x$	0	10	20	30
4	98	91	85	81
6	103	98	92	88
8	112	105	99	93

(a.) Estimate $f_x(10, 6)$ and $f_y(10, 6)$

(b.) Estimate the value of $f(15, 7)$ using the local linearity: $\Delta f \approx f_x \Delta x + f_y \Delta y$.
(Hint: set $\Delta f = f(15, 7) - f(10, 6)$).

8 (13 pts). Let $f(x) = 2x - \ln x$.

(a.6pts). Find the critical point(s) of $f(x)$.

(b. 7pts). Find the global maximum and minimum of $f(x)$ over the interval $1 \leq x \leq 3$ (Show your work.)

9 (13 pts). Let $f(x, y) = x^2 + y^2 - xy - 3y + 30$.

(a. 7pts). Find the critical point (a, b) of the function $f(x, y)$.

(b. 6pts). Apply the test function $D = f_{xx}(a, b)f_{yy}(a, b) - f_{xy}^2(a, b)$ and the sign of f_{xx} to determine whether the critical point is a local minimum,local maximum, or neither.

10 (12 pts). Apply the method of Lagrange multipliers to solve following problem. A company manufactures x units of one item and y units of another. The total cost (in dollars) is given by $C(x, y) = 5x^2 + 2xy + 3y^2 + 800$.

(**a**). If the total number of items (both types combined) is 39, find the minimum cost $C(x, y)$.

(**b**). Estimates the additional minimal cost if the total prodution quota is raised to 41.

11 (12 pts). Find x, y, z such that $\begin{bmatrix} x+y & 2 \\ 4 & 0 \end{bmatrix} = \begin{bmatrix} 6 & x-y \\ 4 & z \end{bmatrix}$.

12 (13pts). Let $A = \begin{bmatrix} 2 & -3 & 4 \\ 0 & 2 & 1 \end{bmatrix}$, $B = \begin{bmatrix} 1 & 2 & 0 \\ 5 & 1 & 2 \end{bmatrix}$, $C = \begin{bmatrix} -3 & 0 & 5 \\ 2 & 1 & 3 \end{bmatrix}$.

Find $2A - 5(B + C)$

13 (12pts). Let matrix $A = \begin{bmatrix} a & 1-a \\ 1+a & -a \end{bmatrix}$, find A^2 (which is the matrix $A \cdot A$.)

14 (13 pts). Solve following system. If the system has infinitely many solution, formulate them using parameter(s). Show your work indetails.

$$\begin{cases} x + 2y + z = 1 \\ 2x - y + z = 2 \\ 3x + y + 2z = 3 \end{cases}$$

15 (13pts). It is given that the inverse $\begin{bmatrix} -1 & 1 & 1 \\ 16 & -14 & -13 \\ 6 & -5 & -5 \end{bmatrix}^{-1} = \begin{bmatrix} 5 & 0 & 1 \\ 2 & -1 & 3 \\ 4 & 1 & -2 \end{bmatrix}$. Make use of the above given information to solve the linear system (write out your formulation):

$$\begin{cases} -x + y + z = 6 \\ 16x - 14y - 13z = 2 \\ 6x - 5y - 5z = 7. \end{cases}$$

Choose One of the following problems. Do both get extra credits.

16 (12pts). A closed economy system consists of three sectors A, B, C. The input–output matrix is given by

\	A	B	C
A	0.2	0.1	0.2
B	0.2	0.3	0.4
C	0.6	0.6	0.4

.

Find the salaries $X = [x, y, z]$ for sectors A, B and C. (You may set the $z = \$30,000$.)

17 (12 pts). The revenue and cost functions for a company are given below in the figure. Should the company produce the 500th item in addtion to a total production of $q = 499$? Show your argument in detail.