Math 205 Final Examination

7:00-8:50PM, December 14, 2005

Instructor:_____ **Class Time:**_____ **Name:**_____

No books or notes are allowed. Please read the problems carefully and do **all** you are asked to do. **You must show your work!** You can use the back page as a scratch paper. Do the problems at the space provided.

Prob.	#1	#2	#3	#4	#5	#6	#7	#8	#9	# 10	# 11	# 12	# 13	#14	#15	#16	Total
Pts																	

1(12 total points). Suppose that the initial deposit of a savings account is $\$5,000$, and the interest is compounded quarterly. If the nominal annual interest rate is 4%, find the balance 10 years since its openning.

2(13 total points). A function $f(x, y)$ satisfies $f(100, 200) = 400$, $f_x(100, 200) = 5$, and $f_y(100, 200) = 8$. Estimate $f(102, 197)$.

3(12 total points). Find the derivatives of the following functions. (Do not simplify)

(a)(4 pts). $y = \dfrac{x}{2+x^2}$.

(b)(4 pts). $f(x) = e^{1+x^2}$.

(c)(4 pts). $f(t) = t\ln(1+2t)$

4(13 total points). Find the global maximum and global minimum of the function $f(x) = x^3 - 12x$ over the interval $-3 \leq x \leq .5$ (Must show your work to get any credit).

5(12 total points). Find the quantity which maximizes the profit if the total revenue and total cost (in dollars) are given by

$$R(q) = 5q - 0.005q^2$$

$$C(q) = 300 + 0.5q$$

where q is quantity and $0 \leq q \leq 1000$.

6(13 total points). Find the indicated partial derivatives of the following functions:

(a)(8pts). Given $f(x, y) = 10x^2 \ln(1 + y^2)$, find

$f_x(x, y) =$

$f_y(x, y) =$

(b)(5 pts). Given $f(x, t) = x^3 - 4x^2t$, find

$f_{xt} =$

7(13 total points). The average cost of production, in thousands of dollars per unit, is $A(q) = q^2 - 12q + 60$, where q is in thousands and $0 \leq q \leq 8$.

(a)(6 pts). Find the total cost function $C(q)$ and the marginal cost function $C'(q)$.

(b)(7 pts) Find the value of q at which the average cost is minimized. What is the minimum average cost?

8(12 total points). Let $f(x, y) = x^2 - 4x + y^2 - 2y + 5$.

(a)(6 pts). Find the critical point of f;

(b)(6 pts). Use the second derivative test involving the value of the function

$$D(x, y) = f_{xx}(x, y)f_{yy}(x, y) - [f_{xy}(x, y)]^2$$

at the critical point to determine if f has a local maximum, a local minimum, or neither at the critical point.

9(13 total points). Use the method of Lagrange multipliers to find the minimum value of $f(x, y) = x^2 + y^2$ subject to the constraint $x + 2y = 12$.

10(12 total points). Suppose that $A + B = 2C$ where

$$A = \begin{bmatrix} x & y \\ y & 8 \end{bmatrix}, \quad B = \begin{bmatrix} y & 3 \\ -x & -6 \end{bmatrix}, \quad \text{and} \quad C = \begin{bmatrix} 5 & 0.5y + 1.5 \\ 4 & 1 \end{bmatrix}.$$

Find x and y.

11(12 total points). Let $A = \begin{bmatrix} -2 & -1 \\ 1 & 0 \end{bmatrix}$, and $B = \begin{bmatrix} 1 & -1 \\ -1 & 1 \\ 0 & 1 \end{bmatrix}$. Perform each of the following operations or indicate that the operation is not defined.

(a)(6pts). $AB =$

(b)(6pts). $BA =$

12(12 total points). For each of the following augmented matrix, check the appropriate box to indicate if the corresponding linear system has a unique solution, has no solution, or has infinitely many solutions.

(a). (5pts) $\left[\begin{array}{ccc|c} 1 & 9 & 2 & 9 \\ 0 & 1 & 3 & -1 \\ 0 & 0 & 0 & 0 \end{array}\right]$

has a unique solution ..... ☐,
has no solution ..... ☐,
has infinitely many solutions ..... ☐.

(b). (5pts) $\left[\begin{array}{cc|c} 1 & 7 & 3 \\ 0 & 1 & 0 \\ 0 & 0 & 0 \end{array}\right]$

has a unique solution ..... ☐,
has no solution ..... ☐,
has infinitely many solutions ..... ☐.

(c). (5pts) $\left[\begin{array}{ccc|c} 1 & -1 & 0 & 1 \\ 0 & 0 & 0 & 0 \\ 0 & 2 & 0 & -2 \\ 0 & 0 & 1 & 0 \end{array}\right]$

has a unique solution ..... ☐,
has no solution ..... ☐,
has infinitely many solutions ..... ☐.

13(13 total points). Use the reduced row-echelon technique to find the inverse of the matrix $A = \begin{bmatrix} 1 & -1 \\ 2 & -4 \end{bmatrix}$.

14(13 total points). Use the fact that the matrix
$A = \begin{bmatrix} -\frac{5}{2} & 2 & -\frac{1}{2} \\ 1 & -1 & 1 \\ \frac{1}{2} & 0 & -\frac{1}{2} \end{bmatrix}$ has the inverse $A^{-1} = \begin{bmatrix} 1 & 2 & 3 \\ 2 & 3 & 4 \\ 1 & 2 & 1 \end{bmatrix}$ to solve the linear system

$$\begin{cases} -\frac{5}{2}x + 2y - \frac{1}{2}z = 1 \\ x - y + z = 0 \\ \frac{1}{2}x - \frac{1}{2}z = -1 \end{cases}$$

No credit will be granted if the given A^{-1} is not used.

15(12 total points). The TOjolobal Mayan Indian community in Southern Mexico has available a fixed amount of land. The proportion, P, of land in use for farming t years after 1935 is modeled with the logistic function

$$P = \frac{1}{1 + 3e^{-0.0275t}}.$$

(a)(4 pts). What proportion of the land was in use for farming in 1935?

(a)(4 pts). What is the long run prediction of this model?

(b)(4 pts). When is the proportion of the land used in farming increasing most rapidly?

16(13 total points). The demand curve of a brand of ice cream is $q = 7200 - 800p$, $2 \leq p \leq 8$. (a)(8 pts). Find the elasticity of demand at the current price of \$4.00. (Hint: q is a linear function of p.)

(b)(5 pts). Should the company increase the price in order to increase the revenue?