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PLANE TRIGONOMETRY
FINAL EXAM
July 27, 2007

There are 10 questions and each worth 20 points. Solve the problems in the space provided. **Show all your work for full points**. Sign and submit your formula sheet with the exam.

1. Express the complex numbers $z_1 = 2\sqrt{3} + 2i$ and $z_2 = 3 - 3i$ in trigonometric form with $0 \le \theta \le 2\pi$.

   Hence find (i) $z_1 z_2$ (ii) $\frac{z_1}{z_2}$

   (iii) $(z_1)^{30}$ and $(z_2)^{24}$ in the form $a + ib$.

2. Find the solutions of the equation $\csc^5\theta - 4\csc\theta = 0$ in the interval $[0, 2\pi]$.

3. Verify the identities by transforming the left side into the right side

(a) $\dfrac{1+\sin^2 5t}{\cos^2 5t} = 2\sec^2 5t - 1$ $\qquad$ (b) $\dfrac{\cos t + \cos 4t + \cos 7t}{\sin t + \sin 4t + \sin 7t} = \cot 4t$

4. Sketch the graph of the equation $y = -2\cos\left(\frac{\pi}{2}x - \frac{\pi}{4}\right) + 1$ by giving the amplitude, period and the phase shift.
   Hence find the number of zero inside $[0, 2\pi]$ of the equation $1 - 2\cos\left(\frac{\pi}{2}x - \frac{\pi}{4}\right) = 0$.

5. To determine the distance between two points A and B, a surveyor chooses a point C that is 370 yards from A and 540 yards from B. If angle $\angle$BAC has measure $47^\circ 30'$, approximate the distance between A and B.

6. Sketch the graph of the polar equation $r = 6\cos\theta - 2\sin\theta$ and identify the curve by changing it to a equation in x and y .

7. (a) Find the polar equation of the line $3y + 2x - 2 = 0$
   (b) Change the polar coordinates to rectangular coordinates.
   (i) $(5, \frac{5\pi}{6})$ (ii) $(-6, \frac{7\pi}{3})$

8. Find the vertex, focus, and the directrix of the parabola $y^2 + 14y + 4x + 45 = 0$. Sketch its graph.

9. Find the points of intersection of the ellipse $x^2 + 4y^2 = 20$ and the line $x + 2y = 6$. Sketch the both curvers in the same plane.

10. Find the vertices, the foci, and the equations of the asymptotes of the hyperbola $\frac{(x-3)^2}{25}-\frac{(y-1)^2}{4}=1$. Sketch its graph.

11. Extra credit problem (10 points)
Sketch the circle $x^2+y^2-2x-6y=0$.