Trigonometry Formulas

$$\sin(u+v) = \sin u \cos v + \cos u \sin v$$

$$\cos(u+v) = \cos u \cos v - \sin u \sin v$$

$$\tan(u+v) = \frac{\tan u + \tan v}{1 - \tan u \tan v}$$

$$\sin(u-v) = \sin u \cos v - \cos u \sin v$$

$$\cos(u-v) = \cos u \cos v + \sin u \sin v$$

$$\tan(u-v) = \frac{\tan u - \tan v}{1 + \tan u \tan v}$$

$$\sin^2 \frac{u}{2} = \frac{1 - \cos u}{2}$$

$$\cos^2 \frac{u}{2} = \frac{1 + \cos u}{2}$$

$$\sin 2u = 2 \sin u \cos u$$

$$\begin{aligned} \cos 2u &= \cos^2 u - \sin^2 u \\ &= 1 - 2\sin^2 u \\ &= 2\cos^2 u - 1 \end{aligned}$$

$$\tan 2u = \frac{2 \tan u}{1 - \tan^2 u}$$

$$\sin a + \sin b = 2 \sin \frac{a+b}{2} \cos \frac{a-b}{2}$$

$$\sin a - \sin b = 2 \cos \frac{a+b}{2} \sin \frac{a-b}{2}$$

$$\cos a + \cos b = 2 \cos \frac{a+b}{2} \cos \frac{a-b}{2}$$

$$\cos a - \cos b = -2 \sin \frac{a+b}{2} \sin \frac{a-b}{2}$$

Trigonometry, MATH 150 NAME _____

May 9, 2005 Rec. Instr. _____

Final Exam Rec. Hour _____

Work as many of the following problems as you can. To assure partial credit, <u>show your work</u>. Calculators <u>are</u> permitted for this exam.

page	1	2	3	4	5	6	7	8	Total
points									

(10 pts) 1. A skater on a circular rink of radius 50 feet spins to a stop after executing a maneuver that has brought her 220 feet in a clock-wise direction along the circumference of the circle. What is the radian measure of the central angle that she has traversed?

(8 pts) 2. Find the radian measure (to four decimal places) of an angle of $27°52'33''$.

(12 pts) 3. Let C be a circle of radius 5, with center at the origin, and let θ be an angle with initial side on the x-axis, and whose terminal side meets C at a point P whose x-coordinate is 3 and whose y-coordinate is negative. Find $\cos\theta$, $\sin\theta$, and $\tan\theta$. (Circle your answers, please.)

(12 pts) 4. The graph of a function $y = a\sin(bx + c)$ is shown below. Find a, b, and c (and circle your answers).

(36 pts) 5. Find the <u>exact</u> values of

a) $\sin(\pi/8)$

b) $\cos\left(\frac{\pi}{3}+\frac{\pi}{4}\right)$

c) $\tan^{-1}(1)$

d) $\sin(\sin^{-1}(\pi/7))$

e) $\cos^{-1}(\cos(5\pi/3))$

f) $\sin(\cos^{-1}(3/5))$

(28 pts) 6. Find all the solutions, in the interval $[0, 2\pi)$, to the equations:

a) $\sin t = -\dfrac{\sqrt{3}}{2}$

b) $\cos(2t) = \dfrac{1}{2}$

c) $\cos\left(2t - \dfrac{\pi}{2}\right) = \dfrac{1}{2}$

d) $\sin t \cos 2t = 0$

(12 pts) 7. A ship steams away from the port, at a steady bearing of $N25°E$. After travelling 10 miles, the captain changes course to $S53°E$, and keeps on in this direction until arriving at an island which lies due east from the port. How far is the island from the port? (Do make a sketch which displays the data for this problem.

(12 pts) 8. How many triangles are there, labelled in the standard way, with

a) $\alpha = 65°, \quad c = 20, \quad a = 12.$

b) $\alpha = 25°, \quad c = 20, \quad a = 16.$

c) $\alpha = 105°, \quad b = 16, \quad c = 3.$

(12 pts) 9. A hyperbola has foci $F(\pm 2, 0)$ and asymptotes $y = \pm x$. Find the vertices of the hyperbola. (It should help to start by sketching a sort of "preliminary graph" from the given data.)

(12 pts) 10. "Classify" the conic: $x^2 + 4y^2 + 2x - 40y + 97 = 0$.
(For example: a wrong answer would be "horizontal parabola"), and find its center.

(10 pts) 11. Find the center and the radius of the circle: $r = 3\sin\theta + 2\cos\theta$.

(12 pts) 12. Describe (in words and/or by a picture) the graphs of the polar equations:

a) $\theta = \pi/2$.

b) $r = 2$.

(12 pts) 13. The polar equation $r = \dfrac{3}{2 - 2\cos\theta}$ is the equation of a parabola. Find the focus, vertex, and directrix of this parabola. (If you can find the focus and the vertex, there should be no problem in finding the directrix.)

(12 pts) 14. "Classify" the conic which is given by the polar equation

$$r = \frac{4}{2 + \sin\theta},$$

and find its center and its vertices.