Name ______________________ Section ______________________

Rec. Instr. ______________________

PLANE TRIGONOMETRY

Final Exam
December 15, 2005

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Below you will find 20 problems, each worth 10 points. Solve the problems in the space provided. When writing a solution to a problem, **show all work**. No books or notes are allowed. **Sign and submit your formula sheet with the exam.**

Problem 1. Let u be an angle in quadrant II, with $\tan u = -\dfrac{5}{12}$. Find the **exact** values of $\sin u$ and $\cos u$.

Problem 2. Verify the identity: $\dfrac{\cot^2 x}{\csc x + 1} = \dfrac{1 - \sin x}{\sin x}$.

Problem 3. Let θ be an angle (in standard position) in quadrant III, whose terminal side is parallel to the line $3x + 4y = 12$. Find the **exact** values of $\sin\theta$ and $\cos\theta$.

Problem 4. Let α and β be two angles in quadrant II, with $\csc\alpha = \dfrac{5}{4}$ and $\sin\beta = \dfrac{15}{17}$. Find the **exact** values of $\sin(\alpha+\beta)$ and $\cos(\alpha+\beta)$.

Problem 5. Find the **exact** value of $\cos\left[2\sin^{-1}\left(\frac{1}{3}\right)\right]$.

Problem 6. Given $180^\circ \le \theta \le 360^\circ$, with $\cos\theta = -\dfrac{119}{169}$. Find the **exact** values of $\sin(\theta/2)$ and $\cos(\theta/2)$.

Problem 7. Find **all** solutions of the equation: $\sin 5t + \sin 3t = 0$. (HINT: Factor LHS.)

Problem 8. Solve $\triangle ABC$, given: $a = 3$, $b = 4$, $\hat{A} = 30^\circ$. (Express all angles in degrees. Round to two decimal places.)

Problem 9. Solve $\triangle ABC$, given: $a = 9$, $b = 4$, $\hat{C} = 60°$. (Express all angles in degrees. Round to two decimal places.)

Problem 10. Solve $\triangle ABC$, given: $a = 6$, $b = 7$, $c = 8$. (Express all angles in degrees. Round to two decimal places.)

Problem 11. Express the complex number $z = 2 - 2i$ in trigonometric form:

$$z = r(\cos\theta + i\sin\theta),$$

with $0 \leq \theta < 2\pi$.

Problem 12. Find the equation of the line that passes through the point $P(1, 1)$, and is perpendicular to the line $x + 5y = 10$.

Problem 13. Find the vertex, focus, and directrix, for the parabola $8(x + 1) = (y - 3)^2$.

Problem 14. Find the equation of the parabola with vertex $V(1, 3)$ and focus $F(1, -1)$.

Problem 15. Find the center, vertices, and foci, for the ellipse:

$$9(x-2)^2 + 25y^2 = 225.$$

Problem 16. Find the equation of the ellipse with vertices $V(\pm 3, 1)$ and foci $F(\pm 2, 1)$

Problem 17. Find the center, vertices, and the asymptotes, for the hyperbola:

$$\frac{(y-1)^2}{9} - \frac{(x+3)^2}{4} = 1.$$

Problem 18. Find the equation of the hyperbola with vertices $V(-1, \pm 2)$ and foci $F(-1, \pm 3)$.

Problem 19. Find the polar equation that has same graph as $(x-1)^2 + (y+1)^2 = 2$. The polar equation should have the form: "$r =$ *expression in* θ."

Problem 20. Let $\mathcal{L}$ be a line, with polar equation $r = \dfrac{6}{2\cos\theta - 3\sin\theta}$.

(a) Find the equation of $\mathcal{L}$ in rectangular coordinates.

(b) Find the x-intercept and the y-intercept of $\mathcal{L}$.