Name: ____________________ **Rec. Instructor:** ____________________

Rec. Time: ____________________

Plane Trigonometry

Final Exam

11:50 a.m. – 1:40 p.m., Dec. 17, 2008

Page 2 (30 pts.)	**Page 3** (30 pts.)	**Page 4** (30 pts.)	**Page 5** (30 pts.)	**Page 6** (30 pts.)	**Page 7** (30 pts.)	**Total** (180 pts.)

Read the Instructions below:

$\Longrightarrow$ No books or notes are allowed.

$\Longrightarrow$ Please read the problems carefully and do **all** you are asked to do.
You must show your work!

$\Longrightarrow$ Each of the 18 problems is worth 10 points. (The total is multiplied by $\frac{10}{9}$ to reach the 200 points of the final.)

Problem 1. Let ϕ be an angle in quadrant IV with $\tan\phi = -\dfrac{5}{12}$. Determine the **exact** values of $\sin\phi$ and $\sec\phi$.

Problem 2. Find the **exact** value of $\tan\left(2\arcsin(\tfrac{3}{4})\right)$.

Problem 3. Solve the triangle ΔABC which has $a = 2$ cm, $b = 5$ cm and $c = 4$ cm. (Express all angles in degrees. Round to two decimal places.)

Problem 3. The triangle ΔABC has $a = 6$ cm, $c = 4$ cm and $\alpha = 60°$. Find the remaining parts: β, γ and b. (Express all angles in degrees. Round to two decimal places.)
Warning: This problem *can* have two solutions!

Problem 5. After touching down, the 2008 Manhattan tornado was heading straight with a velocity of 34 miles/hour in the direction $N60°E$ until it hit our lecture hall 15 minutes later where it (essentially) ended. For a storm-chaser located further south of the lecture hall, the angle between the touch-down point and the lecture hall (i.e., the angle formed by the track of the tornado as seen by the observer) was 80°. How far away from the lecture hall was the storm-chaser?

Problem 6. Find **all** solutions of the equation $\sin(3t) + \sin t = 0$.

Problem 7. Let α and β be two second-quadrant angles with $\csc\alpha = \dfrac{3}{2}$ and $\cos\beta = -\dfrac{1}{3}$. Find the **exact** values of $\tan(\alpha+\beta)$ and the quadrant containing $\alpha+\beta$.

Problem 8. Verify the following identity: $\dfrac{\cos t}{1-\sin t} = \sec t + \tan t$.

Problem 9. Let θ be an angle with $180° \le \theta \le 360°$ and $\cos\theta = \dfrac{127}{225}$. Give the **exact** values of $\sin(\theta/2)$ and $\cos(\theta/2)$.

Problem 10. (a) Express the complex number $z = 4 - 4i$ in the polar form $z = r(\cos\theta + i\sin\theta)$.

(b) Determine the fifth power of z. Write it in polar form with argument in the interval $[0, 2\pi)$.

Problem 11. Find the equation of the line that passes through the point $P(-2, 1)$ and is perpendicular to the line $4x + 5y = -23$.

Problem 12. Determine the type of the graph of the following Cartesian (coordinates are (x, y)) and polar equations (coordinates are (r, θ)). You just have to give the answer here. The possible answers are: line, circle, ellipse, parabola and hyperbola.

equation	type of graph
(a) $-3x^2 - 4x + 5 + 5y + 5y^2 = 4$	
(b) $-3x^2 - 4x + 5 + 5y = 4$	
(c) $r = 3\cos\theta - 4\sin\theta$	
(d) $r = \dfrac{5}{3\cos\theta - 4\sin\theta}$	
(e) $r = \dfrac{5}{3 - 4\sin\theta}$	

Problem 17. Find the vertex, focus, and directrix for the parabola $-4x + 6 = y^2 + 6y + 9$.

Problem 13. Find the equation of the parabola with vertex $V(2, -3)$ and focus $F(2, 1)$ in Cartesian coordinates.

Problem 14. Find the center, vertices, and the asymptotes for the hyperbola:

$$\frac{(x-3)^2}{25} - \frac{(y+1)^2}{9} = 1.$$

Problem 15. An ellipse has the polar equation

$$r = \frac{72}{9 + 3\sin\theta}.$$

Find the foci, the vertices and the equation of the directrix in the corresponding Cartesian coordinate system.

Problem 16. The largest planet of our solar system is Jupiter which travels in an elliptical orbit that has eccentricity 0.0488 and a major axis of length 10.41 AU. Find the maximum and minimum distances between Jupiter and the sun (located in one of the two foci of the ellipse).

Problem 18. Find the polar equation that has the same graph as the circle $(x+4)^2+(y-1)^2 = 17$. The polar equation should have the form "$r =$ *expression in* θ."