Paulhus (8), Trachelman (8)

Name: Key
Recitation Instructor:
Recitation Day and Time:

College Algebra – Exam 3

April 7, 2009

Page 1 24 pts.	Page 2 22 pts.	Page 3 30 pts.	Page 4 24 pts.	TOTAL 100 pts

Directions: You will find 10 problems listed below. The point value of each problem is given in parentheses. Please show all your work neatly and box your final answers. No notes or books are allowed. You have one hour to complete this exam.

1. (8 points) Solve and write your final answer in interval notation: $|3x - 6| < 6$

$-6 < 3x - 6 < 6$

$-6 < 3x - 6$ and $3x - 6 < 6$

$0 < 3x$ $\quad$ $3x < 12$

$0 < x$ and $x < 4$

$\boxed{(0, 4)}$

2. (8 points) Set up and solve an inequality to answer the following question. **You must define any variables used.** Company A charges \$72 per hour to install granite countertops, while company B charges a fixed cost of \$240 plus \$60 per hour for the same services. For what period (or interval) of time is it cheaper to hire company A?

Let t = # of hours.

Company A's charge: $72t$

Company B's charge: $60t + 240$.

$72t < 60t + 240$.

$12t < 240$.

$t < 20$ hours.

$\boxed{0 < t < 20 \text{ hours}}$

3. (8 points) Find a polynomial with a single zero at $x = -3$ and $x = 5$, and a double zero at $x = 2$. Write your answer in the form $a_n x^n + \cdots + a_1 x + a_0$ (in other words, in descending powers of x).

$(x+3)(x-5)(x-2)^2$

$= (x^2 - 2x - 15)(x^2 - 4x + 4)$

$= x^4 - 4x^3 + 4x^2$

$\quad - 2x^3 + 8x^2 - 8x$

$\quad\quad - 15x^2 + 60x - 60$ $= \boxed{x^4 - 6x^3 - 3x^2 + 52x - 60}$

~~[illegible]~~ (8), Shreshtha (8)
Sorokin

4. (12 points) Consider the polynomial $p(x) = x^4 - 2x^3 + x^2 - 8x - 12$.

(a) Using the Rational Zero Theorem, list all possible rational zeros of $p(x)$.

Possible rational roots given by $\frac{\text{factors of constant}}{\text{factors of lead. coefficient}}$

$= \boxed{\pm 1, \pm 2, \pm 3, \pm 4, \pm 6, \pm 12}$

(1) (b) Given that $x = -1$ and $x = 3$ are zeros of $p(x)$, find the other zeros of $p(x)$.

Poly. division:

$$\begin{array}{r} x^3 - 3x^2 + 4x - 12 \\ x+1 \overline{\big) x^4 - 2x^3 + x^2 - 8x - 12} \\ -(x^4 + x^3) \\ -3x^3 + x^2 \\ -(-3x^3 - 3x^2) \\ 4x^2 - 8x \\ -(4x^2 + 4x) \\ -12x - 12 \\ -(-12x - 12) \\ 0 \end{array}$$

$$\begin{array}{r} x^2 \quad + 4 \\ x-3 \overline{\big) x^3 - 3x^2 + 4x - 12} \\ -(x^3 - 3x^2) \\ 4x - 12 \\ -(4x - 12) \\ 0 \end{array}$$

(2) Synth. Division:

$$\begin{array}{r|rrrrr} -1 & 1 & -2 & 1 & -8 & -12 \\ & \downarrow & -1 & 3 & -4 & 12 \\ \hline 3 & 1 & -3 & 4 & -12 & 0 \\ & \downarrow & 3 & 0 & 12 & \\ \hline & 1 & 0 & 4 & 0 & \end{array}$$

$x^2 + 4 = 0$

$x^2 + 4 = 0$

$(x+2i)(x-2i) = 0$

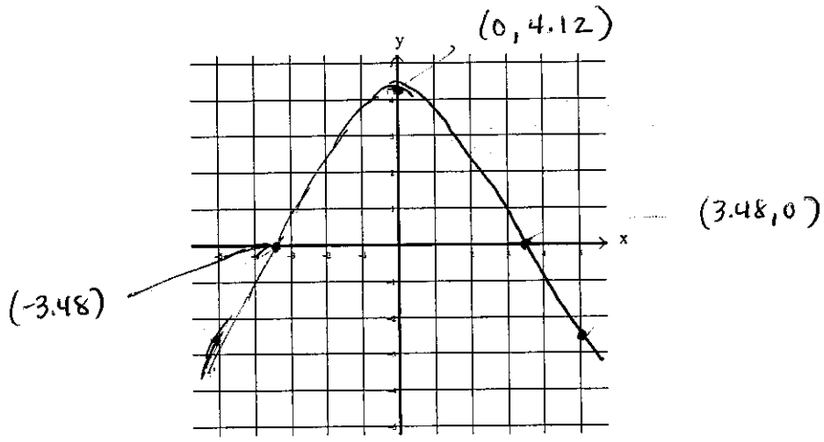
$(x+2i)(x-2i) = 0$

$\boxed{x = \pm 2i}$ Answer.

(c) Factor the polynomial $p(x)$ completely. In other words, write $p(x)$ as a product of linear factors.

$p(x) = (x+2i)(x-2i)(x+1)(x-3)$

5. (a) (6 points) Using your graphing calculator, graph $f(x) = .005x^4 - .4x^2 + 4.12$ over the following intervals: $-5 \le x \le 5$ and $-5 \le y \le 5$. Label all intercepts.



(b) (4 points) What features of the graph of $f(x)$ are missing in the above viewing window? Explain.

(1) End behavior: degree of $f(x)$ is even
Lead. coeff. is positive
As $x \to \infty$, $f(x) \to \infty$; as $x \to -\infty$, $f(x) \to \infty$.

(2) 2 more x-intercepts.

Pigno (6), Burckel (6), Cupra (4)

6. (7 points) Using **long division**, find the slant asymptote of the following rational function:
$r(x) = \dfrac{2x^3 + 3x^2 + 7x + 4}{x^2 + x + 1}$

$$
\begin{array}{r}
2x + 1 \\
x^2 + x + 1 \overline{\big) 2x^3 + 3x^2 + 7x + 4} \\
-(2x^3 + 2x^2 + 2x) \\
\hline
x^2 + 5x + 4 \\
-(x^2 + x + 1) \\
\hline
4x + 3
\end{array}
$$

$\boxed{y = 2x + 1}$

7. (12 points) Consider the rational function $r(x) = \dfrac{(5x+6)}{(2x+1)(x-4)}$.

(a) Find the vertical asymptotes of $r(x)$.

$2x + 1 = 0$ $\quad$ $x - 4 = 0$

$\boxed{x = -1/2}$ $\quad$ $\boxed{x = 4}$

(b) Find the horizontal asymptote of $r(x)$.

$y = 0$, since degree of numerator < degree denominator

(c) Find the zeros of $r(x)$.

$5x + 6 = 0$

$\boxed{x = -6/5}$

(d) Find the y-intercept of $r(x)$.

$r(0) = \dfrac{6}{-4} = \dfrac{-3}{2}$ $\quad$ $\boxed{(0, \frac{-3}{2})}$

8. (11 points) Our text discusses an example in which the temperature, in degrees Fahrenheit, of a person during an illness is given by the function $T(t) = \dfrac{98.6t^2 + 4t + 98.6}{t^2 + 1}$, where t is the number of hours after the onset of the illness.

(a) What is $T(7)$? $\boxed{99.16^\circ F}$

(b) What is the horizontal asymptote of $T(t)$? $\boxed{T = 98.6}$ (since degree of numerator = degree denom.)

(c) What does your answer in part (b) imply about body temperature?

As time goes on, body temp. goes back to normal, 98.6°F.

Kamunski (6) ~~[illegible]~~ (6), Natarajan (4)
Moore

9. (6 points) Solve the quadratic inequality $x^2 - 9 > 0$.

$(x-3)(x+3) > 0$.

$+$ | -3 | $-$ | 3 | $+$

test $x=-4$: $(-4-3)(-4+3) = (-7)(-1) = 7$

test $x=0$: $(0-3)(0+3) = (-3)(3) = -9$

test $x=4$: $(4-3)(4+3) = (1)(7) = 7$

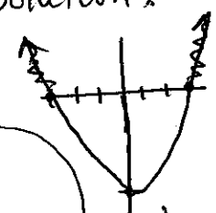
$\boxed{(-\infty,-3) \cup (3,\infty)}$

Solution!

10. (18 points) Consider the polynomial $p(x) = x^3 - 12x - 16$.

(a) Using the leading-term test, describe the end behavior of $p(x)$.

Odd degree
positive lead. coefficient, so as $x \to \infty$, $p(x) \to \infty$
$x \to -\infty$, $p(x) \to -\infty$



graphical solution

(b) Given that $x = -2$ is a zero of $p(x)$, find the other zeros of $p(x)$. Please show all work (calculator answers are not allowed).

(1)

$$\begin{array}{r} x^2 - 2x - 8 \\ x+2 \overline{\big) x^3 + 0x^2 - 12x - 16} \\ -(x^3 + 2x^2) \\ \hline -2x^2 - 12x \\ -(-2x^2 - 4x) \\ \hline -8x - 16 \end{array}$$

Remainder = 0.

$(x^2 - 2x - 8) = 0$

$(x-4)(x+2) = 0$

$\boxed{x=4,\ x=-2}$

or (2) Synthetic Div:

$$\begin{array}{r|rrrr} -2 & 1 & 0 & -12 & -16 \\ & \downarrow & -2 & 4 & 16 \\ \hline & 1 & -2 & -8 & 0 \end{array}$$

$x^2 - 2x - 8$

(c) What is the y-intercept of $p(x)$?

$p(0) = -16$. $\boxed{(0,-16)}$

(d) Using the **Remainder Theorem**, find $p(2)$.

(1)

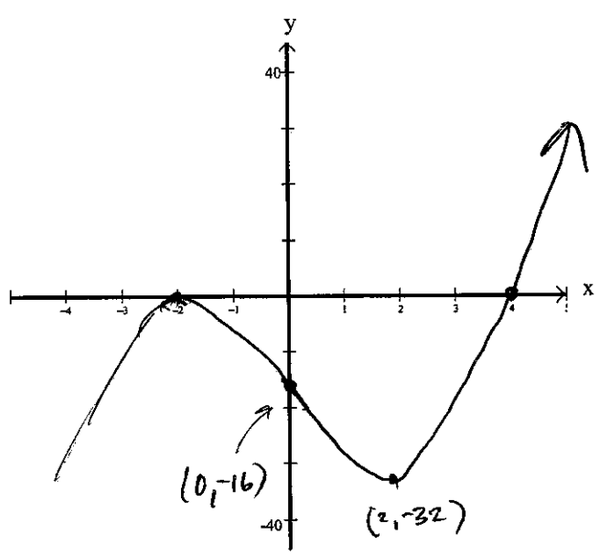
$$\begin{array}{r|rrrr} 2 & 1 & 0 & -12 & -16 \\ & \downarrow & 2 & 4 & -16 \\ \hline & 1 & 2 & -8 & -32 \end{array}$$

$\boxed{p(2) = -32}$

(2) Long division

$$\begin{array}{r} x^2 + 2x - 8 \\ x-2 \overline{\big) x^3 + 0x^2 - 12x - 16} \\ -(x^2 - 2x^2) \\ \hline 2x^2 - 12x \\ -(2x^2 - 4x) \\ \hline -8x - 16 \\ -(-8x + 16) \\ \hline -32 \end{array}$$

$= p(2)$

(e) Using the information from parts (a)-(d), sketch $p(x)$ in the coordinate system below.

* $x = -2$ is a repeated root.