**Name:**
**Recitation Instructor:**
**Recitation Day and Time:**

# College Algebra – Exam 1

February 3, 2009

| Page 1 26 pts. | Page 2 26 pts. | Page 3 24 pts. | Page 4 24 pts. | TOTAL 100 pts |
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**Directions:** You will find 14 problems listed below. The point value of each problem is given in parentheses. Please show all your work neatly and box your final answers. No notes or books are allowed. You have one hour to complete this exam.

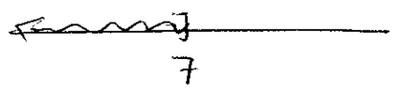
1. (6 points) Classify the following statements as True or False. If false, explain why.

   (a) $\mathbb{Q} \subseteq \mathbb{N}$ False; there are several elements of $\mathbb{Q}$ not contained in $\mathbb{N}$, such as $\frac{3}{5}, \frac{2}{3}$, etc.

   (b) $\frac{3}{5} \in \mathbb{Q}$ True

2. (6 points) Write in interval notation and then graph on a number line: $\{x \mid x \leq 7\}$

   $(-\infty, 7]$

3. (6 points) Multiply and display answer in standard form: $(k-5)(k^2+12)$

   $k^3 + 12k - 5k^2 - 60$

   $\boxed{k^3 - 5k^2 + 12k - 60}$

4. (8 points) Factor completely: $10x^2 + 19x - 15$

   $\boxed{(2x+5)(5x-3)}$

5. (6 points) Factor into linear factors: $64x^2 - 289$

$$\boxed{(8x-17)(8x+17)}$$

6. (6 points) Simplify completely: $\dfrac{(4a^3b^2)^3}{(2a^4)^2}$

$$\frac{4^3a^9b^6}{2^2a^8} = \frac{64a^9b^6}{4a^8}$$

$$= \boxed{16ab^6}$$

7. (6 points) Simplify completely (no decimals allowed): $\sqrt[3]{54a^3b^4}$

$$= \sqrt[3]{27 \cdot 2 \cdot a^3 \cdot b^3 \cdot b}$$

$$= \boxed{3ab\sqrt[3]{2b}}$$

8. (8 points) Find the least common denominator and simplify: $\dfrac{1}{x^2-3x+2} + \dfrac{1}{x-2} + \dfrac{2}{x-1}$.

LCD: $(x-2)(x-1)$

$$\frac{1+1(x-1)+2(x-2)}{(x-2)(x-1)} = \frac{\cancel{1}+x-\cancel{1}+2x-4}{(x-2)(x-1)}$$

$$= \boxed{\frac{3x-4}{(x-2)(x-1)}}$$

9. (6 points) Express in terms of a single radical (no decimals): $5\sqrt{28}+\sqrt{63}$

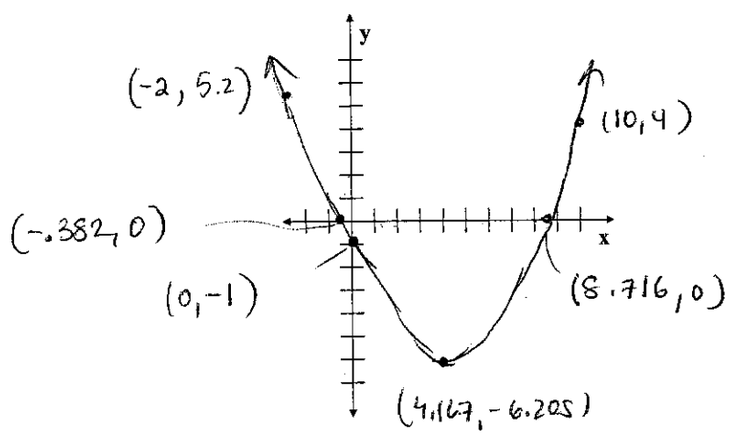
$$5\sqrt{28}+\sqrt{63} = 5\sqrt{4\cdot 7} + \sqrt{9\cdot 7}$$
$$= 5\cdot 2\sqrt{7} + 3\sqrt{7}$$
$$= 10\sqrt{7} + 3\sqrt{7} = \boxed{13\sqrt{7}}$$

10. (6 points) Simplify completely, using only positive exponents in your answer: $(6a^2b^{-5})(2a^{-3}b^4)$.

$$(6\cdot 2)(a^{2-3})(b^{-5+4}) = 12a^{-1}b^{-1} = \boxed{\frac{12}{ab}}$$

11. (12 points)

(a) On the grid below, graph $f(x) = .3x^2 - 2.5x - 1$ over the following intervals: $-2 \le x \le 10$ and $-7 \le y \le 7$. Along with your graph, label axes and intercepts appropriately for full credit.




(b) Find the x-intercepts of this graph using your TI-83/84 and briefly explain which buttons you used. 2nd + trace → choose zero. Hit enter after determining left/right bounds; hit enter to guess.

(c) Find the y-value corresponding to $x = 2.785$ using your graphing calculator. Round answer to the nearest thousandth.

2nd + trace → choose value;

$\boxed{y = -5.636 \text{ when } x = 2.785}$

12. (8 points) Simplify completely: $\dfrac{x^2-y^2}{m+n} \div \dfrac{x-y}{m+n}$

$$\frac{\cancel{(x-y)}(x+y)}{\cancel{(m+n)}} \cdot \frac{\cancel{(m+n)}}{\cancel{(x-y)}} = \boxed{x+y}$$

13. (8 points) Given the points $(-7,9)$ and $(6,4)$, answer the following:

$x_1\ y_1 \quad x_2\ y_2$

(a) What is the distance between the two points? Leave answers in simplified radical form; no decimals please. $d = \sqrt{(x_1-x_2)^2+(y_1-y_2)^2}$

$$d = \sqrt{(-7-6)^2+(9-4)^2}$$
$$= \sqrt{(-13)^2+(5)^2} = \sqrt{169+25} = \boxed{\sqrt{194}}$$

(b) What is the midpoint of the two points?

$$\text{mdpt} = \left(\frac{x_1+x_2}{2}, \frac{y_1+y_2}{2}\right) = \left(\frac{-7+6}{2}, \frac{9+4}{2}\right) = \boxed{\left(\frac{-1}{2}, \frac{13}{2}\right)}$$

14. (8 points)

(a) Given that $a^3+b^3=(a+b)(a^2-ab+b^2)$ holds for all numbers $a$,$b$, find the factorization of the **difference** of two cubes.

Consider $A^3-B^3$, which is $(A)^3+(-B)^3$. Set $a=A$ and $b=-B$ in above formula. Then it follows that

$$A^3-B^3 = (A)^3+(-B)^3 = (A+(-B))(A^2-(A)(-B)+(-B)^2)$$
$$= (A-B)(A^2+AB+B^2).$$

(b) Using your answer from part (a), factor $125y^3-64$.

Set $A = 5y$, $B = 4$. Then $125y^3-64 = (5y)^3-(4)^3$

$$= (5y-4)((5y)^2+(5y)(4)+(4)^2)$$
$$= \boxed{(5y-4)(25y^2+20y+16)}$$