Name:
Recitation Instructor:
Recitation Day and Time:

College Algebra – Exam 3

November 11, 2008

Page 1 23 pts.	Page 2 26 pts.	Page 3 25 pts.	Page 4 26 pts.	TOTAL 100 pts

Directions: You will find 12 problems listed below. The point value of each problem is given in parentheses. Please show all your work neatly and box your final answers. No notes or books are allowed. You have one hour to complete this exam.

1. (7 points) Solve and write your final answer in interval notation: $-4 \leq 8 - 2x < 4$

$-4 \leq 8 - 2x$ and $8 - 2x < 4$

$-12 \leq -2x$ and $-2x < -4$

$6 \geq x$ and $x > 2$

$x \leq 6$ and $x > 2$

$\boxed{(2, 6]}$ Final Answer

2. (8 points) Set up and solve an inequality to answer the following question. You must define any variables used. Company A charges \$25 per hour to install ceramic tiles, while company B charges a fixed cost of \$300 plus \$15 per hour for the same services. For what period (or interval) of time is it cheaper to hire company B?

Let t = # of hours

$25t$ is what A charges

$15t + 300$ is what B charges

$15t + 300 < 25t$

$-10t < -300$

$\boxed{t > 30 \text{ hrs}}$

(For any job that takes longer than 30 hours).

3. (8 points) Find a polynomial with a single zero at $x = -2$ and at $x = 4$, and a double zero at $x = 2$. Write answer in the form $a_n x^n + \cdots + a_1 x + a_0$ (in other words, multiply out completely).

$(x+2)(x-4)(x-2)^2$

$= (x^2 - 2x - 8)(x^2 - 4x + 4)$

$= x^4 - 2x^3 - 8x^2 - 4x^3 + 8x^2 + 32x + 4x^2 - 8x - 32$

$= \boxed{x^4 - 6x^3 + 4x^2 + 24x - 32}$ One possible answer

4. (12 points) For the polynomial $p(x) = 3x^3 + x^2 - 48x - 16$, answer the following.

(a) Using the Rational Zero Theorem, list all possible rational zeros of $p(x)$.

Possible Rational Zeros: $\dfrac{\text{Factors of constant}}{\text{Factors of leading coefficient}}$

$\dfrac{\pm 1, \pm 2, \pm 4, \pm 8, \pm 16}{\pm 1, \pm 3}$ $\quad \pm 1, \pm 1/3, \pm 2, \pm 2/3, \pm 4, \pm 4/3, \pm 8, \pm 8/3, \pm 16, \pm 16/3$

(b) Given that $x = 4$ is one zero of $p(x)$, find the other zeros of $p(x)$.

$$\begin{array}{r|rrrr} 4 & 3 & 1 & -48 & -16 \\ & \downarrow & 12 & 52 & 16 \\ \hline & 3 & 13 & 4 & 0 \end{array}$$

$(3x^2 + 13x + 4) = (3x+1)(x+4) = 0$

$\boxed{x = -1/3 \text{ or } x = -4}$

OR

$$\begin{array}{r} 3x^2 + 13x + 4 \\ x - 4 \overline{\big) 3x^3 + x^2 - 48x - 16} \\ -(3x^3 - 12x^2) \\ \hline 13x^2 - 48x \\ -(13x^2 - 52x) \\ \hline 4x - 16 \\ -(4x - 16) \\ \hline 0 \end{array}$$

$\rightarrow$ Another factor of $p(x)$

$3x^2 + 13x + 4 = 0$

$(3x+1)(x+4) = 0$

$\boxed{x = -1/3 \text{ or } x = -4}$

(c) Factor the polynomial $p(x)$ completely. In other words, write $p(x)$ as a product of linear factors.

$\boxed{p(x) = (x-4)(3x+1)(x+4)}$

5. (6 points) Given $p(x) = 2x^3 - 4x^2 + 32x - 64$, use the **Remainder Theorem** to find $p(-4)$.

Divide $2x^3 - 4x^2 + 32x - 64$ by $x+4$ and find the remainder:

Method 1:

$$\begin{array}{r|rrrr} -4 & 2 & -4 & 32 & -64 \\ & \downarrow & -8 & 48 & -320 \\ \hline & 2 & -12 & 80 & -384 \end{array}$$

$\boxed{\text{So, } p(-4) = -384}$

Method 2:

$$\begin{array}{r} 2x^2 - 12x + 80 \\ x + 4 \overline{\big) 2x^3 - 4x^2 + 32x - 64} \\ -(2x^3 + 8x^2) \\ \hline -12x^2 + 32x \\ -(-12x^2 - 48x) \\ \hline 80x - 64 \\ -(80x + 320) \\ \hline \boxed{-384} \end{array}$$

6. (8 points) Solve the polynomial inequality $x^3 + x^2 - 4x - 4 < 0$.

$x^2(x+1) - 4(x+1) < 0$

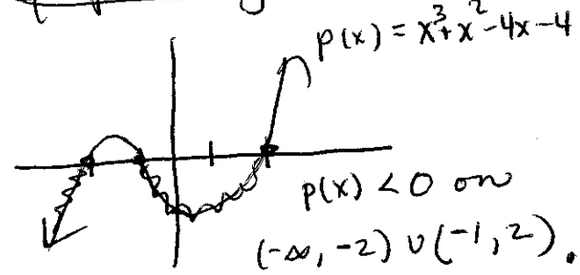
$(x+1)(x^2 - 4) < 0$

$(x+1)(x-2)(x+2) < 0$

$\ominus$ | test $x = -3$ | -2 | $\oplus$ test $x = -1.5$ | -1 | $\ominus$ test $x = 0$ | 2 | $\oplus$ test $x = 3$

$\boxed{\text{Answer: } (-\infty, -2) \cup (-1, 2)}$

Graphically:



$p(x) < 0$ on $(-\infty, -2) \cup (-1, 2)$.

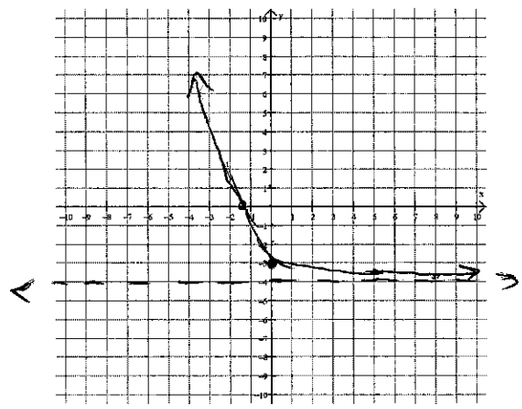
7. (7 points) Using **long division,** ~~find the slant asymptote of~~ simplify the following rational function:

$$r(x) = \frac{2x^3 + 3x^2 + 7x + 3}{x^2 + x + 3}$$

$$\begin{array}{r} 2x + 1 \\ x^2+x+3 \overline{\big) 2x^3 + 3x^2 + 7x + 3} \\ -(2x^3 + 2x^2 + 6x) \\ \hline x^2 + x + 3 \\ -(x^2 + x + 3) \\ \hline 0 \end{array}$$

$\boxed{r(x) = 2x + 1}$

8. (a) (5 points) Using your graphing calculator, graph $f(x) = e^{-x} - 4$ over the following intervals: $-10 \le x \le 10$ and $-10 \le y \le 10$. Label all intercepts and asymptotes.



(b) (4 points) Using your graphing calculator, find $f(5)$ and $f(8)$.

$f(5) \approx -3.9933$

$f(8) \approx -3.9997$

(c) (3 points) What value does $f(x)$ approach as $x \to \infty$?

$\boxed{f(x) \to -4}$

9. (6 points) Find the inverse of the function $f(x) = x^3 - 4$.

$y = x^3 - 4$

$x = y^3 - 4$

$x + 4 = y^3$

$\boxed{\sqrt[3]{x+4} = y}$

$\boxed{\sqrt[3]{x+4} = f^{-1}(x)}$.

10. (8 points) Consider the polynomial $p(x) = -5x^3 + 3x^2 - 14$.

(a) What is the degree of $p(x)$?

3

(b) What is the leading coefficient of $p(x)$?

-5

(c) Using the leading-term test, describe the end behavior of $p(x)$.

Since leading coefficient is ~~odd~~ negative and degree is odd,

as $x \to \infty$, $p(x) \to -\infty$, and as $x \to -\infty$, $p(x) \to \infty$.

11. (12 points) Consider the rational function $r(x) = \dfrac{(3x+1)(x+1)}{x^2 - 8x + 12}$.

(a) Find the vertical asymptotes of $r(x)$.

$(x^2 - 8x + 12) = 0 \quad (x-6)(x-2) = 0 \quad \boxed{x = 6 \text{ and } x = 2}$

(b) Find the horizontal asymptote of $r(x)$.

$y = \dfrac{\text{lead coeff numerator}}{\text{lead coeff denominator}} \quad y = \dfrac{3}{1} \quad \boxed{y = 3}$

(c) Find the zeros of $r(x)$.

$(3x+1) = 0$ and $(x+1) = 0 \quad \boxed{x = -1/3 \text{ and } x = -1}$

(d) Find the y-intercept of $r(x)$.

$r(0) = \dfrac{1}{12} \quad \boxed{(0, \tfrac{1}{12})}$

12. (6 points) Suppose that after a lapse of t minutes, the number of a certain kind of bacteria can be estimated by the function $N(t) = 2000(2^{t/20})$. How many bacteria will there be after 40 minutes?

$$N(40) = 2000(2^{40/20})$$
$$= 2000(2^2)$$
$$= 2000(4)$$
$$= \boxed{8000 \text{ bacteria}}$$