Name:
Recitation Instructor:
Recitation Day and Time:

College Algebra – Exam 3
April 8, 2008

Page 1 22 pts.	Page 2 26 pts.	Page 3 30 pts.	Page 4 22 pts.	TOTAL 100 pts

Directions: You will find 13 problems listed below. The point value of each problem is given in parentheses. Please show all your work neatly and box your final answers. No notes or books are allowed. You have one hour to complete this exam.

1. (7 points) Solve and write your final answer in interval notation: $|2x - 3| > 7$

$2x-3 > 7$ or $2x-3 < -7$

$2x > 10$ or $2x < -4$

$x > 5$ or $x < -2$

Interval notation:

$\boxed{(-\infty, -2) \cup (5, \infty)}$

2. (8 points) Set up and solve an inequality to answer the following question. You must define any variables used. Firm A charges \$70 per hour for its services, while Firm B charges a fixed cost of \$200 plus \$50 per hour for the same services. For what length of time is it cheaper to hire firm A?

Let t = # of hours

$70t < 50t + 200$

$20t < 200$

$t < 10$

For $t < 10$ hours, it is cheaper to hire firm A (of course, $t > 0$ as well).

3. (7 points) Given $p(x) = 2x^4 + 3x^3 + 16x^2 + 27x - 18$, list all possible rational roots of $p(x)$. (You do not have to find the actual roots.)

$\dfrac{\text{Factors of constant term}}{\text{Factors of leading coefficient}}$: $\dfrac{\pm 1, \pm 2, \pm 3, \pm 6, \pm 9, \pm 18}{\pm 1, \pm 2}$

Rational Zeros Theorem

List: $\pm 1, \pm 2, \pm 3, \pm 6, \pm 9, \pm 18,$
$\pm \frac{1}{2}, \pm \frac{3}{2}, \pm \frac{9}{2}$

4. (8 points) Find a polynomial with a single zero at $x = 1$ and at $x = 3$, and a double zero at $x = -2$. Write answer in the form $a_n x^n + \cdots + a_1 x + a_0$ (in other words, multiply out completely).

$$(x-1)(x-3)(x+2)^2 = (x^2-4x+3)(x^2+4x+4)$$
$$= x^4+4x^3+4x^2-4x^3-16x^2-16x+3x^2+12x+12$$
$$= \boxed{x^4-9x^2-4x+12}$$

{ Any constant (non-zero) multiple of this solution, as long as it's multiplied out, would work also. For instance, $10x^4-90x^2-40x+120$ would be another solution. }

5. (10 points) Given the polynomial $p(x) = x^4 - 3x^3 + 11x^2 - 27x + 18$, and the fact that $x = 1$ and $x = 2$ are zeros,

(a) Find all the zeros of $p(x)$.

1]	1	-3	11	-27	18
	↓	1	-2	9	-18
2]	1	-2	9	-18	0
	↓	2	0	18	
	1	0	9	0	

$$x^2+9=0$$
$$x^2=-9$$
$$x = \pm 3i$$

$\boxed{\text{Zeros: } x=1,\ x=2,\ x=3i,\ x=-3i}$

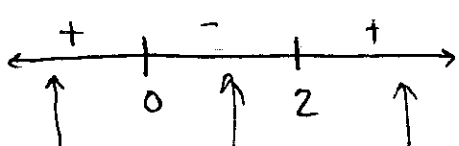
(b) Factor the polynomial $p(x)$ completely. In other words, write $p(x)$ as a product of linear factors.

$p(x) = (x-1)(x-2)(x-3i)(x+3i)$.

6. (8 points) Solve the inequality: $x^2 - 2x < 0$.

Method 1:

$x(x-2) < 0$



test $x=-1$: $(-1)(-3)$ → 3 positive

test $x=1$: $(1)(-1)$ → -1 negative

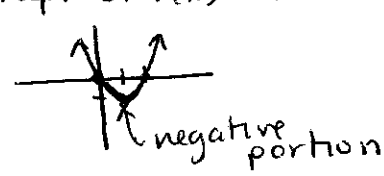
test $x=3$: $(3)(1)$ → 3 positive

→ $x^2-2x<0$ on the interval $(0,2)$ and nowhere else. $\boxed{\text{Solution: } (0,2)}$.

Method 2: $f(x) = x^2-2x+1-1$ ✓ complete the square

$f(x) = (x-1)^2-1$

This shows that x^2-2x is a parabola with vertex $(1,-1)$. The zeros of $f(x)$ are $x=0$ and $x=2$. The graph of $f(x)$ is negative on $(0,2)$, and nowhere else:



7. (6 points) Use the Remainder Theorem to find $f(-3)$ when $f(x) = 3x^3 - 2x + 1$.

If -3 is substituted for x in the polynomial f(x), then f(-3) is the remainder that would result when dividing f(x) by (x+3). You may use long or synthetic division. Here is the answer with synthetic division.

-3	3	0	-2	1
	↓	-9	27	-75
	3	-9	25	-74

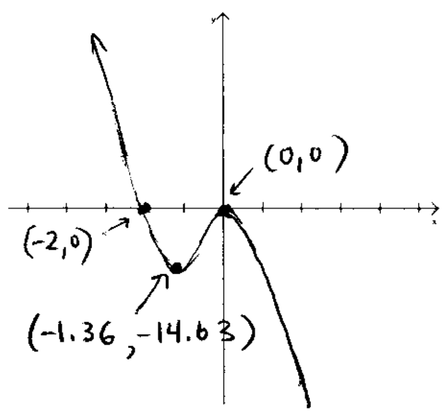
$\boxed{f(-3) = -74}$

8. (8 points) Using **long division**, find the quotient and remainder: $\dfrac{3x^3 + 2x + 5}{x + 1}$

$$
\begin{array}{r}
3x^2 - 3x + 5 \\
x+1 \,\big)\, 3x^3 + 0x^2 + 2x + 5 \\
-(3x^3 + 3x^2) \\
-3x^2 + 2x \\
-(-3x^2 - 3x) \\
5x + 5 \\
-(5x + 5) \\
0
\end{array}
$$

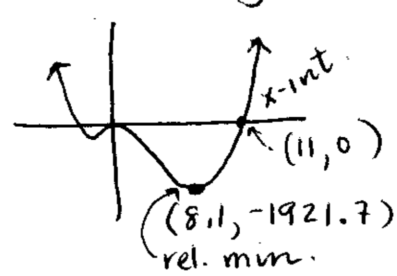
Quotient: $3x^2 - 3x + 5$
Remainder: 0

9. (a) (5 points) Using your graphing calculator, graph $f(x) = x^4 - 9x^3 - 22x^2$ over the following intervals: $-5 \le x \le 5$ and $-50 \le y \le 50$. Label all intercepts and relative extrema.



(b) (5 points) What features of the graph of $f(x) = x^4 - 9x^3 - 22x^2$ are missing in this window? Explain.

This is a 4th degree polynomial with positive leading coefficient. So, as $x \to \infty$, $f(x) \to \infty$. Therefore, we are missing an x-intercept and a relative minimum in this window (we need one more turning point).



10. (6 points) Find the inverse of the function $f(x) = x^3 - 7$.

$y = x^3 - 7$

$x = y^3 - 7$

$x + 7 = y^3$

$\sqrt[3]{x+7} = y$

$\boxed{f^{-1}(x) = \sqrt[3]{x+7}}$

11. (8 points) Consider the polynomial $p(x) = -x^4 + 5x^2 - 4$.

(a) What is the degree of $p(x)$? 4

(b) Using your graphing calculator, determine the exact number of turning points on the graph of $p(x)$. 3

(c) Using the leading-term test, describe the end behavior of $p(x)$.

p(x) is of even degree, and the leading coefficient is negative. By the leading-term test, as $x \to \infty$, $p(x) \to -\infty$, and as $x \to -\infty$, $p(x) \to -\infty$.

12. (8 points) Consider the rational function $r(x) = \dfrac{(2x+1)(x+1)}{x^2 - 6x + 8}$.

(a) Find the vertical asymptotes of $r(x)$.

Set denominator $= 0$: $(x^2 - 6x + 8) = 0$

$(x-4)(x-2) = 0$

$\boxed{\begin{array}{l} x = 4 \\ x = 2 \end{array}}$ Vertical Asymptotes.

(b) Find the horizontal asymptote of $r(x)$.

Numerator has degree 2; Lead. coeff of numerator = 2

Denominator has degree 2 Lead. coeff of denominator = 1

$\boxed{y = 2}$

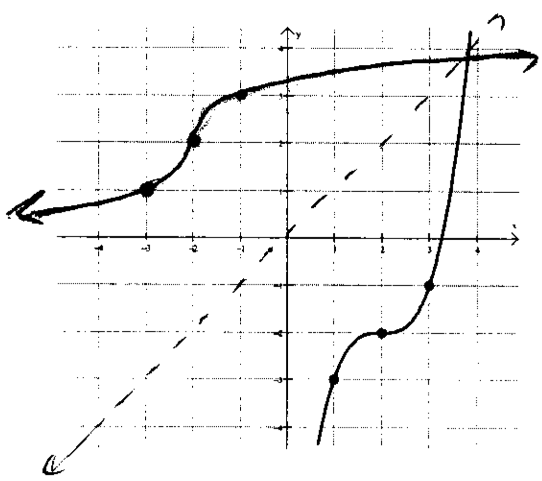
(c) Find the zeros of $r(x)$.

$(2x+1)(x+1) = 0$ $\boxed{x = -1/2 \text{ and } x = -1}$

(d) Find the y-intercept of $r(x)$.

$r(0) = \dfrac{(0+1)(0+1)}{8} = \dfrac{1}{8}$ $\boxed{(0, \tfrac{1}{8})}$

13. (6 points) Given the graph of $f(x)$ below, graph $f^{-1}(x)$ on the same set of axes.



· Points on $f(x)$:

(1, -3)

(2, -2)

(3, -1)

Switch x & y to get points on $f^{-1}(x)$:

(-3, 1)

(-2, 2)

(-1, 3)

Reflect f(x) over line y = x.